Unit Outline: Seventh Grade Math

Introduction: This is a unit that focuses on developing understanding and applying proportional relationships. Students will learn to reason with proportionality through the use of tables, diagrams, verbal descriptions, equations, and graphs. Students will use proportional relationships to solve real-world problems.

Unit Topic and Length:

The topic of this unit is proportional relationships. The unit consists of five lessons, each of which is approximately 50 minutes.

The unit is built upon students' prior knowledge of constructing and analyzing tables, and use of equations to describe relationships of quantities. Students will be extending and reinforcing their conceptual understanding of fractions to solve real-world problems. This is not the only unit for seventh-grade math; therefore, it should not be the introductory unit for proportional relationships. In addition, this unit has been developed to extend students' concept of ratio, rate, and proportional reasoning. Included in each lesson is a list of prerequisite skills and suggested techniques needed to activate students' prior knowledge.

There are several assessment tools included in this unit:

- an initial assessment to be administered prior to the start of the unit to reveal students' misconceptions and gaps;
- exit slips (short assessments) to be administered at the end of each lesson to inform the teacher of students' understanding of the daily objective; and
- a summative assessment for the end of the five-lesson unit, to be used as a measure of students' learning.

Common Core Learning Standards:

<u>Focus Standards</u>

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

7. RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

c. Represent proportional relationships by equations.

d. Explain what a point *(x, y)* on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r)* where *r* is the unit rate.

7.EE.4 Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Standards for Mathematical Practice

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics

MP.5 Use appropriate tools strategically

MP.6 Attend to precision.

Foundational Standards

6. RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities**.**

6.RP.2: Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship.

6.RP.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

b. Solve unit rate problems including those involving unit pricing and constant speed.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

d. Use ratio reasoning to convert measurement units; manipulate and transform unit appropriately when multiplying or dividing quantities.

6.NS.1 Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem.

Big Ideas/Enduring Understandings:	**Essential Question:**
● Develop the thinking ability to reason with proportions. ● Many types/varieties of situations in life depend on or require the application of	● How do we use proportional relationships in solving real-world problems?

ratios and proportional reasoning.	
Language Goals: • Students will compare and contrast proportional relationships using adjectives (and comparative suffixes *er, ier, -est, iest*). • Students will use oral language to discuss and debate using cooperative grouping. • Students will justify and reason orally and in writing. • Students will read and write using sentence-frame structure. • Students will develop mathematical vocabulary using visual representations (charts, graphic organizers).	**Content Objectives**: • Compute, explain, and apply unit rates. • Calculate the missing values in a ratio table. • Find the missing value of a proportion. • Determine whether two ratios are in a proportional relationship. • Translate from one representation of a proportional relationship to another. • Read and interpret linear graphs (Quadrant I). • Represent proportional relationships as an equation ($y = mx + 0$), where m is the proportionality constant. • To understand qualitative predictions and comparisons using proportional reasoning. • Understand that a variable can represent a quantity that can change often in a relationship to another quantity. • Compare and contrast ratios, graphs.
Explicit Support for English Language Learners: • Activate prior knowledge to make connections to new information. • Act-it-out to make use of kinesthetic mode of learning. • Sentence frames provide structure in developing written and oral language. • Read aloud gives opportunities for equal participation. • Wait time allows sufficient time to	**Prerequisite Skills:** • Simplify and understand fractions. • Familiarity with metric/customary system. • Operational skills with fractions. • Identify equivalent fractions. • Identify ratios. • Convert fractions to percent and percent to fractions. • Understand ratios (part to part; part to whole).

process and develop a response. • Visuals and manipulatives provide opportunities for concrete representation of abstract concepts.	• Use ratio table to find a missing value.

Assessment Evidence and Activities:

<u>Initial Assessment</u>

- Unit rate
- Finding missing value in a proportion
- Completing a ratio table
- Comparing rates
- Interpreting ratio graphs (Quadrant 1)
- Writing equations with the use of variables
- Operational skill with fractions

<u>Formative Assessment</u>

Lesson 1 Exit Slip

- Understanding of unit rates

Lesson 2 Exit Slip

- Finding missing value in a proportion

Lesson 3 Exit Slip

- Comparing rates

Lesson 4 Exit Slip

- Interpreting proportionality in graphs

Lesson 5 Exit Slip

- Comparing rates and applying to real-life scenario

<u>Summative Assessment</u>

- Unit rate
- Finding missing value in a proportion
- Completing a ratio table
- Comparing rates

- Interpreting ratio graphs (Quadrant 1)
- Writing equations with the use of variables
- Operational skill with fractions

Learning Plan and Activities:

- Using think-pair-share and whole-class discussion provides students with opportunities to reflect on the meaning of a ratio and a unit rate.
- Through the use of ELL strategies, students will learn to determine whether two ratios are proportional. Group-work activities, think-pair-share, and independent practice will allow the students to discuss mathematical concepts as well justify their thinking orally and in writing.
- Discussing and understanding the meaning of the numbers used in a ratio, students develop strategies to compare ratios. Through the use of think-pair-share and group activities, students will compare and manipulate ratios from real life scenarios.
- Linear graphs and proportional relationships are used a tool to have students further their understanding of what proportionality is (and is not), as well as increasing their ability to make predictions and comparisons using proportional reasoning.
- Activate students' prior knowledge to read and interpret a proportionality graph in a real-life problem using the think-pair-share strategy. Students work collaboratively to apply their understanding of unit rate in comparing the costs of two phone plans, and write equations to express each plan.

Resources:

EngageNY.org. A Story of Ratios: A curriculum overview for Grades 6-8. 2013. http://www.engageny.org/resource/grades-6-8-mathematics-curriculum-map

Cramer, K. and T. Post. Connecting Research To Teaching Proportional Reasoning. *Mathematics Teacher*, 86(5), 404-407, May 1993.

Kersaint, G., et al. *Teaching Mathematics to English Language Learners*. New York: Routledge, 2008.

Brown, J., and A. Sanford. *RTI for English Language Learners: Appropriately Using Screening and Progress Monitoring Tools to Improve Instructional Outcomes*. Portland State University, March 2011.

Thompson, S., and S. Vaughn. *Research-Based Methods of Reading Instruction for English Language Learners*. New York: Association for Supervision & Curriculum Development, 2007.

The Common Core Standards Writing Team. Progressions for the Common Core State Standards: *6-7, Ratios and Proportional Relationships,* 2011. http://commoncoretools.files.wordpress.com/2012/02/ccss_progression_rp_67_2011_11_12_corrected.pdf

Unit: Seventh Grade Math

Initial Assessment

Name: _____ **Date:** _____

The students are selling cookies at a school fair. Sara is selling three chocolate chip cookies for $2.49.

a) If you wanted to buy one cookie from Sara, how much would it cost? _____

b) How much would it cost to buy 20 of Sara's cookies? _____

Show your work and explain your reasoning in the space provided below:

c) Choose three of the missing values in the table below. In the space provided below the table, show how you found the values.

Number of cookies	1	3	20	100	50	150			525
Cost (in dollars & cents)		$2.49					$145.25	$190.90	

d) John is selling seven chocolate chip cookies for $5.39. Who offers the better price, Sara or John? _____

Explain why:

e) Sara earned $49.80 in a quarter of an hour. If she continues to sell cookies at this rate, how many cookies would she sell in an hour and a half? _____

Show your work in the space provided below:

f) The graph below represents the price of Trisha's chocolate chip cookies. How much would it cost to buy 30 of Trisha's cookies?_____

Justify your answer in the space provided below the graph.

g) Trisha would like to find the price (p) of any number (n) of cookies. In the space below, write an equation to find the price of any number of cookies.

h) At the end of the sale, Trisha asked her mom what she should do with the leftover $16\frac{1}{2}$ lbs. of cookies. Her mom suggested packing them into $\frac{3}{4}$ lb. bags. How many $\frac{3}{4}$ lb. bags can she make? _____

Justify your answer in the space below.

Lesson 1: How Fast Can You Text?

Mathematical Goals:

- Students compute, explain, and apply ratios and unit rates.
- Students will learn and use different ways of representing rates and ratios.

Language Goals:

- Students will read, write, listen to, and speak about ratio and rate using pictures, numbers, and written descriptions.
- Students will justify their reasoning using informal and formal mathematical language.
- Students will use adjectives with the comparative suffixes (*-er, -er, -est, -iest*) in orally and written language.

Materials:

- Chart paper
- Calculators (optional)
- Two-color counters, colored cubes (optional)
- Handout #1: Worksheet
- Handout #2: Exit Slip

Common Core Learning Standards:

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Standards for Mathematical Practice:

No listed numbers.

Common Issues	Suggested Questions and Prompts
Simplifying ratio and rate	• What does it mean to simplify? • What happens to the numerator and denominator when you simplify? • Use manipulatives to demonstrate that your ratio is in simplest terms when you cannot divide the numerator and denominator by the same number. For example, begin with 4 yellow counters and 6 red counters. Ask students: *Can we divide the yellow and red counters into the same number of groups?*

	The yellow and red counters can both be divided into 2 groups. Identify how many are in each group. (There are 2 yellow and 3 red in each group). Because they cannot be further divided into smaller groups, the ratio is in simplest terms.
Ignoring the units	• Ask students: *Is $4 per gallon the same as 4 gallons per $1?*
Differentiating between the part-to-part and part-to-whole ratios	• In a class, 2 out of 3 students are boys. – What does the 2 represent? *[Boys]* – What does the 3 represent? *[Whole]* In this case, it is considered part to whole. – If 2 out of 3 students are boys, how many are girls? *[1 out of 3]* – The ratio of boys to girls would be 2 to 1. *[Part-to-part]* • Create situations in which students convert a part/part ratio to a part/whole and vice versa. For example, if the ratio of red marbles to blue marbles is 3:5, then the ratio of red marbles to all marbles (red and blue) is 3:8. Ask students to tell you the ratio of blue marbles to all marbles.

Vocabulary Instruction:

It is favored that ELLs explore mathematical concepts and then attach related language to it. Mathematics vocabulary often arises from experience and the investigations. However, unfamiliar words that may impact ELLs' ability to follow the lesson should be taught explicitly. Addressing content vocabulary gives ELLs the opportunity to identify words and then be able to place them in context as they learn them. This lesson presents some vocabulary that teachers should address when appropriate.

Focus Vocabulary Definitions

- **ratio** – a multiplicative comparison between two quantities. It can be represented as *a:b, a to b,* or *a/b.*
- **unit** – a quantity used as a standard of measurement
- **rate** – the ratio of two different measurements with different units
- **unit rate** – a ratio that indicates how many units of one quantity there are for every one unit of the second quantity (e.g., miles per hour, price per pound, students per class, heartbeats per minute)

Tiered Vocabulary

Tier I	Tier II	Tier III
cellphone	comparison pound miles distance gallons statistic	ratio proportion qualitative quantitative

Suggested Lesson Outline (with Embedded Support for ELLs):

- Introduction to ratios: Scenario #1 and discussion
- Independent work: Scenario #2 and discussion
- Silent think-pair-share: Scenario #3, and discussion
- Extra practice groups or pairs: Handout #2
- Exit slip: Handout #3

Prerequisite Skills

(Use the prerequisite skills and techniques as a Do Now, entrance slip, or as the prior lesson's exit slip to assess students' readiness for this lesson.)

Prerequisite Skills	Techniques
• Equivalent fractions	• What is the meaning of the 2 and 3 in the fraction $^{2}/_{3}$? Draw a picture to represent $^{2}/_{3}$. Next, draw a picture of $^{4}/_{6}$. What do you notice about the two diagrams? • Draw a bar model of the fraction $^{3}/_{4}$. Next cut each piece in half. What is the new fraction? • Are $^{3}/_{4}$ and $^{6}/_{8}$ equal? Why or why not? • Create a table beginning with the fraction, and then skip-count by the numerator and

<table>
<tr><td></td><td>denominator.
<table><tr><td>2</td><td>4</td><td>6</td><td>8 ...</td></tr><tr><td>3</td><td>6</td><td>9</td><td>12 ...</td></tr></table></td></tr>
<tr><td>• Simplify fractions</td><td>• Teacher will use fraction strip models to represent a fraction being simplified. For example, what is another way to show $^{6}/_{8}$ using fewer model pieces?
• $\frac{4}{6} = \frac{2}{?}$
How can we find the value of the question mark?</td></tr>
<tr><td>• Ratio language</td><td>• Use the words during explicit teaching, discourse.
• Have word bank available.
• Develop a brainstorm chart that is continuous and can be used throughout the lessons.</td></tr>
<tr><td>• Different ways of representing ratios</td><td>• Given the image below, represent the ratio using a colon, the word “to,” and a fraction.
★★★○○○○○</td></tr>
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<u>Lesson Description</u>

Distribute ***Handout #1: Worksheet*** to each student.

Ask a student to read Scenario #1 aloud while the rest of the class reads along silently.

Next, ask:

- *What is the question being asked? (Paraphrase the question.)*
- *What does “two out of three” mean?*
- *How can we use the information that two out of three middle school students have a cellphone to answer the question?*
- *What other information is needed to answer the question?*

Activating prior knowledge scaffolds knowledge which ELLs possess in order to make connections to new concepts/ information.

- *If the answer is not a whole number, what does that mean? For example, if there are 28 students in the class, there would be 18 $\frac{2}{3}$ students with a cellphone. How do we interpret the situation in a real-life situation?*
- *What is your solution? How did you arrive at your solution?*
- *What are the different strategies that can be used to arrive at the answer?* (Record students' strategies on chart paper.)

You might want to represent the ratio by having the class act it out.

Next, tell students they are going to work independently to answer the question in Scenario #2 on the handout.

> *Scenario #2: Maria said that she is able to use her cellphone to text 45 words in 15 seconds. How many words can she text in 1 second?*

Written charts are visuals that maximize ELLs' learning in math and language development. It is one avenue to increase communications, whether is through using charts to speak or using them for writing.

Give the class five minutes to read the scenario and answer the question on the handout. Monitor the students and identify three to share their strategies with the class. As the students share each strategy, record the strategy on chart paper to be displayed in the room.

Strategies observed may include:

- Dividing: 45 texts / 15 seconds = 3 texts per second
- Setting up the unit proportion and finding the value of the "?"'
- Draw a diagram: Draw 45 items, and make groups of 15.
- Use a ratio table:

divide by 3 divide by 5

Number of texts	45	15	3
Number of seconds	15	5	1

If necessary, provide explicit instruction. Before moving on to the next scenario, the above four strategies should be charted and posted in the classroom.

Explicit instruction provides clear and easy procedures to follow.

Next, ask students to read along silently as you read aloud Scenario #3 on the handout:

> *Scenario #3: José told Maria that he texts faster than she does. José can text 48 words in 12 seconds. Who do you think can text faster, José or Maria? Justify your answer.*

Have students solve this problem using the silent think-pair-share method. (Explain the strategy if necessary.)

Give the students eight minutes to complete the problem. Once the eight minutes are up, instruct the students to exchange handouts with their partner. While partners are working, monitor the class and note students who are using different strategies, specifically highlighting students using the unit rate. Ask those students to share those strategies during the classroom discussion.

Think-pair-share supports ELLs in becoming comfortable with the development of oral skills. It gives them a greater chance to participate and practice modalities of listening and speaking.

Facilitate the class discussion by giving students the following prompts at the appropriate times:

- *Raise your hand if you think Maria texts faster. Raise your hand if you think José does. Explain how you know who texts faster.*
- *How can unit rates be helpful in finding out who is the person who texts faster?*
- *Why is finding unit rates important?*
- *In what other situations can unit rates be applied?*

Providing wait time generates greater participation by ELLs by allowing them to process their thinking and develop a response.

Students will now work independently on unit rates in Part 2 on Handout #1. (Suggestion: Modify the numbers in the problems on the handout to provide additional practice.)

Lead a closing discussion on the following questions. Refer to the charts created throughout the lesson to help answer them.

- What is an example of a rate? A ratio? (Students' responses might be summarized using a T-chart.)
- How do we calculate a rate?
- Look at the tables obtained by different groups and observe any discrepancies. If there are any, discuss them with the class, having students justify their answers and provide an explanation of their work.
- What is a rate?
- Create another real-world situation in which a unit rates are useful.

Assessment

Distribute ***Handout #2: Exit Slip*** to each student explain that they are to complete it independently. When time is up, collect the exit slips, making sure students' names appear on them. Use the exit slips to assess students' comprehension of the lesson.

Lesson 1: How Fast Can You Text?

Handout #1:

Name: _____ **Date:** _____

Part 1

Scenario #1: Statistics show that two out of three middle school students have a cell phone. According to this statistic, how many students in this class should have a cell phone? _____

Represent this information as a ratio: _____

Work space:

Scenario #2: Maria said that she is able to use her cell phone to text 45 words in 15 seconds. How many words can she text in 1 second? _____

Work space:

Silent Think-Pair-Share

Scenario #3: José told Maria that he texts faster than she does. José can text 48 words in 12 seconds. Who do you think can text faster, José or Maria? Justify your answer.

My Answer and Justification	**My Partner's Feedback and Justification**
Check one: ☐ Maria texts faster. ☐ José texts faster. *Supporting work:* *Justification:* _____ _____ _____ _____	*Check one:* ☐ I agree ☐ I disagree *Supporting work:* *Justification:* _____ _____ _____ _____

<u>Part 2</u> – ***Practice applying and finding the unit rate.***

1) Diana travelled 385 miles in 7 hours. What is her average speed in miles per hour?

   _____

2) Is each of the following considered a unit rate? Explain why or why not.

   - 60 calories per serving: *Yes* _____ *or No* _____

     Explain: _____

     _____

     _____

     _____

   - 300 miles in 5 hours: *Yes* _____ *or No* _____

     Explain: _____

     _____

     _____

     _____

   - 5 packs of gum for $1: *Yes* _____ *or No* _____

     Explain: _____

     _____

     _____

     _____

   - 17 heartbeats in 15 seconds: *Yes* _____ *or No* _____

     Explain: _____

     _____

     _____

     _____

3) Charlie bought a $2\frac{1}{2}$ pound bag of oranges for $3.75. What is the cost of one pound of oranges? _____

4) Mrs. Sweets provided cookies for her class of 24 students. She purchased 72 cookies to make sure that each student receives the same number of cookies. Complete the table below to determine the number of cookies per student.

Number of cookies	72				
Number of students	24	12	6	2	1

5) Manuel worked 5 days a week, and 6 hours each day. That week he earned $337.50. What was his hourly rate of pay?

6) Use unit rate to determine which of the following is the best deal:

- $4.08 for a 16-ounce box of cereal
- $4.50 for an 18-ounce box of cereal,
- $5.39 for a 22-ounce box of cereal?

Lesson 1: How Fast Can You Text?

Handout #2: Exit Slip

Name: _____ **Date:** _____

Word Bank

ratio	***unit***	***rate***	***unit rate***

1. Describe what a unit rate is.

   _____

   _____

   _____

   _____

2. Using two of the math vocabulary words in the word bank, give three real-life scenarios of unit rate.

   _____

   _____

   _____

   _____

   _____

3. How much more expensive per ounce is a 15-ounce bottle of mango juice for $3.45 than a 32-ounce bottle of mango juice for $5.12?

   Work Space:

   Answer: _____

Lesson 2: Missing a Text

Mathematical Goals:

- Students will be able to determine if two ratios are proportional.
- Students will be able to find the missing value in a proportion.

Language Goals:

- Students will justify their mathematical thinking verbally and in writing.
- Students will prove ratios as proportions using graphic organizer.

Materials:

- Handout #1: Scenario #1
- Handout #2: Fraction and Cross-Product Strategies Worksheet
- Chart paper to record student's responses throughout lesson
- Manipulatives, such as colored counters (optional)

Common Core Learning Standards:

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics.

MP.5 Use appropriate tools strategically.

MP.6 Attend to precision.

Common Issues	Suggested Questions and Prompts
Operational errors in simplifying ratio and rate problems due to the lack multiplication tables.	Teacher can choose to: • Explain in what way multiplication and addition are alike. • Use flashcards. Post one multiplication fact (single-digit) on the classroom door for a week so students get to see

	the fact on their way in and out of the room. Change the fact the next week.
Uncertainty of understanding fractions	Teacher can choose to: • Use visuals to represent the numerator and denominator. • Use fraction strips throughout the unit.
Ignoring the consistency of units in proportion	Teacher can choose to: • Chart examples of ratios and the correct units (i.e., 2 boys out of 5 boys, or 2 boys to 5 girls). • Have a list of different units.

Vocabulary Instruction:

It is suggested that ELL students explore mathematical concepts and then connect related language to them. The vocabulary can be developed from everyday life experiences and the class investigations. However, unfamiliar words that may impact ELLs' ability to follow the lesson should be taught explicitly. Addressing content vocabulary gives ELLs the opportunity to identify words and then be able to place them in context as they learn them. This lesson presents some vocabulary that teachers should address when appropriate. Words in this lesson which might potentially challenge ELLs are shown below.

Focus Vocabulary Definitions

- **ratio** – a multiplicative comparison between two quantities. It can be represented as *a:b, a to b,* or *a/b*.
- **unit** – a quantity used as a standard of measurement
- **proportional relationship** – one in which two quantities vary directly with one another
- **proportiona**l – comparative

Tiered Vocabulary

Tier I	Tier II	Tier III
results	survey equivalent simplest form	GCF (greatest common factor) numerator

	constant	denominator ratio proportional fraction product variable

Suggested Lesson Outline (with Embedded Support for ELLs):

- Summarize previous lesson outcome.
- Introduce Scenario #1.
- Explicit teaching and note-taking
- Independent practice
- Discussion
- Closure: Whole-class summarizing

Prerequisite Skills

(Use the prerequisite skills and techniques as a Do Now, entrance slip, or as the prior lesson's exit slip to assess students' readiness for this lesson)

Skill	Technique
Define and identify a ratio.	• Have the definition charted and visible for students to be able to refer back to, along with the pictorial representation for students to make connections. • Display the different ways to represent a ratio (e.g., 2 to 5; 2:5; 2/5).
Understand equivalent fractions	• Make fraction strips available (cutouts). • Use fraction manipulative pieces.
Find the GCF	• Reference chart with the different strategies to find the GCF: *Strategy 1* Listing the common factors of numbers *8: 1, 2, 4, 8*

	6: 1, 2 , 3, 6 so the GCF for 8 and 6 is 2. *Strategy 2* The factor tree

Lesson Description

Begin the lesson by quickly summarizing the previous day's exit slip, starting with the meaning of *ratio*.

Activating prior knowledge is an effective scaffold for ELLs schema. In addition, it helps make connections to other concepts or new information.

Tell students to use notes/ charts from previous lesson to continue to build knowledge with today's lesson.

Say: *Today we will try to determine if two ratios are proportional or not, and to learn the different strategies for finding out if the set of ratios are proportional.*

Have a separate visible chart where the math terms (*ratio, proportional, numerator, denominator*) will be written and used throughout the lesson. Continue to add words to the chart as the lessons progress.

To engage students in thinking mode, ask the following questions. A class discussion will follow. Chart the questions to create a visual aid for all students to read.

The use of visuals, such as a chart that includes vocabulary words with pictures, can help ELL students have access during class discourse.

- How many of you have heard the word "proportional"?
- In which situations have you encountered the word "proportional"?
- What does it mean to be proportional?

At this point, chart three or four different student responses. If responses are incorrect, chart one misconception and refer to it as the lesson flows. If students have not provided any responses, present the word "proportional" in context. Give students a copy of ***Handout #1: Scenario #1***. Explain that the handout has two parts. Tell them to read Part I of the handout silently for two minutes, but not to go beyond that point.

Wait time allows ELLs to process thinking and develop a response. Greater participation by learners occurs.

Next, read Part 1 aloud as students read along silently. Tell them to notice where the word "proportional" is placed in the scenario. If necessary, repeat the three questions that you asked and charted before distributing the handouts to clarify the phrase "whether the two ratios are proportional." Give students a synonym for word "survey" (poll or questionnaire) to bring access to the scenario.

Cooperative learning, such as Think-Pair-Share supports ELL oral language development using the modality of listening and speaking.

Allow students five minutes to practice the think-pair-share strategy. Tell the students to think independently about the proportionality of both age groups and then turn to their partner and share what they think. Monitor students' discussions with the focus of choosing some responses that will lend to the whole-class discussion's insight into the concept of proportion.

Make discussion questions visible. Then, read them aloud, pausing after each question (about eight seconds) to give students enough thinking time to respond:

- *What do you understand from the situation? What do we know?*
- *What does 48 out of 64 mean? 72 out of 96?*
- *What is the question asking?*
- *How can we determine if the two ratios are proportional?*
- *Share your thoughts about solving this scenario.*

Scaffolding questions provide ELLs with linguistic fluency in spontaneous discussions. It helps them to build on the knowledge they possess to make conjectures.

Allow students to share their responses.

Next, explicitly model determining if two ratios are proportional: Use the visuals of fractions to represent the fractions in Scenario #1. These visuals or picture models should have been prepared ahead of time. Again, there is only one scenario on this handout.

You might begin by showing the one strategy (fraction strategy) with charting the two ratios. First, write them as 48:64 = 72:96. (Remind the class of the unit for these numbers, 48 students out a total group of 64 students). Explain that these ratios will be written in fraction form as a way to find if the two are proportional, because ratios are not easy to work with.

For students in the early stages of English language acquisition, prompt students to use non-linguistic responses (e.g. point to, show me, circle) as you circulate to monitor and assess their work.

Tell students: *The first number in the first ratio (48) is the numerator, and the second number in the first ratio (64) is the denominator.*

Bring attention to the equal sign between the two ratios. Now that the ratios are rewritten as fractions (48/64 and 72/96), students should know that the fraction needs to be reduced to its lowest terms. Therefore, the second step is to reduce both fractions and see whether they have the same value. If they do, then they are proportional.

Say: *To see if these two fractions are proportional, we need to reduce them to their simplest form.*

Direct the students to the explicit model of how to reduce a fraction to its simplest form.

Ask students: *What do we need to do to find if these two fractions are proportional? [We need to reduce them to their simplest form.]*

How do we reduce them to their simplest form? [We need to find the greatest common factor of each fraction (GCF), and then divide the numerator and denominator by this factor.]

At this point, if some students do not recall how to find the GCF, use two to three minutes of explicit instruction mentioned above. A GCF chart can be prepared ahead of time to serve as a reference.

Find the GCF with the help of the students. In this case, the GCF is 16. Divide 48/64 by 16/16 (which is equal to 1, but is another way to write it). The resulting fraction will be equivalent to the original fraction.

Tell the class: *When we divide 48/64 by 16/16, the resulting fraction is ¾, which means that 48/64 is equivalent to ¾.*

Follow the same procedure to find the GCF for the second fraction (72/96). It is also ¾.

To prove whether both simplified fractions are proportional, compare the fractions to see if they have the same value or are equal to each other.

Say: *Since ¾ equals ¾, we know that the ratio 48/64 is proportional to the ratio 72/96.*

Because the word "proportional" is used again, ask students to explain what it means to be proportional. If needed, go back to any misconceptions charted at the beginning of the lesson.

Repetition of vocabulary through context reinforces the usage and develops understanding of new concepts.

Provide students with two sets of ratios to apply the strategy to: 12 girls out of 20 girls, and 21 girls out of 35. Another set of ratios can be 16 rotten pears out of 56 pears, and 10 rotten apples out of 35 apples. (Chart these ahead of time.) Give students five minutes to work independently. Monitor students as they begin to use the strategy (fraction). Quickly share the steps and solutions to both ratios.

Tell students to record the fraction strategy on Part 2 of Handout #1, which can be used as a reference for the strategies being reinforced in this lesson.

Introduce the cross-product strategy—at this point.

Written text and visual representation help ELLs to refer back to content. It can be used as a tool to maximize interaction and build a language-rich environment.

Say: *Suppose that in another scenario there is a piece of information missing. In this case, we can use another strategy called the cross-product strategy.*

Provide Scenario #2 (charted ahead of time):

- Maria took 20 minutes to drive 4 miles. If the speed is constant, then how many minutes will it take for Maria to drive 12 miles?

Questions you can ask to elicit responses are:

- *What do we know about Maria?*
- *What does it mean "speed is constant"?*
- *How can we determine the number of minutes Maria will take to travel 12 miles?*

Model how to use the cross-product algorithm. Explain to students that this method can be efficient if the problem is set up correctly. Be aware when setting the fractions that the units must match across to each other. For instance, the fraction is 20 minutes/4 miles = *x* minutes/12 miles. The units for the numerators are minutes, whereas the units for denominators are miles.

Explicit instruction provides clear procedures to follow as ELLs learn not only a new skill/strategy but also language usage.

Continue to show the algorithm: 20 minutes will be cross-multiplied with 12 miles, so the value is 240. Then *x* minutes (an unknown variable is named *x*) is multiplied by 4 miles, which is *x* and 4 miles. At this point, demonstrate how we divide to find the missing value. Divide 240 by 4, which equals 60.

Say*: Since the missing value is 60, to prove that both fractions are proportional, we will use multiplication: 20 multiplied by 12 equals 240, which is 60 multiplied by 4.*

In this case, both values are the same. Tell students that, in this case, it will take Maria 60 minutes to drive 12 miles. Both fractions are equal to each other, which makes them proportional.

Have the students record the cross-product strategy on Part 2 (Note-Taking) of Handout #1. They can record one of the examples and the steps taken to solve for the missing value.

Next, give students two fraction problems to practice what has been taught:

- 3 gallons in 7 hours; __ gallons in 20 hours
- 8 inches in 25 minutes; inches in __ minutes

Give students five minutes to work independently. Monitor the class to check for understanding as students work. Go over the problems as a whole class.

To reinforce both strategies distribute ***Handout #2: Fraction and Cross-Product Strategies Worksheet*** to students. Students will be using both strategies. Give them 10 minutes

Monitoring, during group or independent tasks, serves as a tool to confirm an ELL's need for additional instructional support.
As you monitor to assess, prompt learners at early stages of English language acquisition to use non-linguistic responses such *as point to, show me,* etc.

to work on the problems. If students are not completely finished when time is up, tell them to do so for homework.

Lead the class in a final discussion on the day's lesson. Suggested prompts are:

- Describe one concept that was learned today.
- Explain how to determine if two ratios are proportional.
- Why are proportional relationships important in our everyday life?

Chart students' responses, which will help them to learn new concepts for the next lesson.

Exit slips allow ELLs to demonstrate their strengths through the process of choosing one problem. ELLs will get the opportunity to express their strengths through writing. It also provides the teacher with feedback to make additional instructional decisions.

Assessment:

Distribute ***Handout #3: Exit Slip*** to each student. Allow them 10 minutes to choose and solve one of the two problems on the handout independently. When time is up, collect the exit slips, making students' names appear on them. Use the exit slips to assess students' comprehension of the lesson.

Lesson 2: Missing a Text

Handout #1: Scenario #1

Name: _____ Date: _____

Directions:

1. Read the following scenario.
2. Think-pair-share: Discuss whether the two groups are proportional.
3. Be ready to share with the whole class.

Scenario #1

Part 1

A survey was taken to find out which age group texted more while class is in session. The results show that in the group of 14-year-olds, 48 students out of 64 students texted. In the other age group, 15-year-olds, it was 72 students out of 96.

- Are these two groups proportional?
- Are these ratios proportional?

My Answer and Reasoning	**My Partner's Answer and Reasoning**
I think that these two groups (ratios) are ☐ proportional ☐ not proportional because _____ _____ _____ _____ _____ _____	My partner thinks that these two groups (ratios) are ☐ proportional ☐ not proportional because _____ _____ _____ _____ _____ _____

<u>Part II</u> – *Note-Taking*

Describe two different strategies used to determine whether the ratios are proportional.

Strategy #1:	**Strategy #2:**
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____
_____	_____

Lesson 2: Missing a Text

Handout #2: Fraction and Cross-Product Strategies Worksheet

Name: _____ **Date**: _____

Part I

Directions: Use the *fraction strategy* to prove whether each pair of ratios are proportional or not.

A. ***Proportional or not proportional?*** 8:3 = 48:18 Show your work:	**B.** ***Proportional or not proportional?*** 5:8 = 12:20 Show your work:

Part II

Directions: Read the following scenarios. Find the missing value using the *cross-product strategy.*

Scenario #1 A cookie recipe says to use 3 cups of flour and 2 eggs. If you wanted to increase the recipe and use 9 cups of flour, what would the ratio of flour to eggs be?	***Scenario #2*** Anthony is typing a paper that is 390 words long. He can type 30 words per minute. How long will it take Anthony to type the paper?

Part III

Directions: Read Scenario #3 below and solve it. Explain your mathematical thinking using math terms (*ratio, proportion, per, product*).

Scenario #3

An automobile travels 176 miles on 8 gallons of gasoline. How far can it go on a tankful of gasoline if the tank holds 14 gallons?

Show your work.

Lesson 2: Missing a Text

Handout #3: Exit Slip

Name: _____ **Date:** _____

Word Bank

ratio	*proportional*	*compare*	*numerator*	*denominator*	*fraction*

Directions: Choose one of the two scenarios below and solve it. Show all your work and explain the question of the scenario chosen. Include at least three mathematical terms in your writing.

Scenario #1

Cell phone company A is promoting a special deal for the month of September. It offers 24 pictures with text for $32. Another company—company B—is offering 135 pictures with text for $180. Are these two promotions proportional to each other? How can you determine which company offers the better deal? Show your work. Try to use words from the word bank above to explain which company has the better deal.

_____

_____

_____

_____

_____

_____

Scenario #2

José and David's teacher gave the class a book to read at home. David read 40 pages of a book in 50 minutes. José read 68 pages in 80 minutes. David says that the ratios are proportional. Is this true?

_____

_____

_____

_____

_____

_____

Lesson 3: Do More Girls or Boys Use Their Cellphones to Get Online?

Mathematical Goals:

- Students will compare and contrast ratios.
- Students will understand qualitative predictions and comparisons using proportional reasoning.

Language Goals:

- Students will compare and contrast proportional relationships using adjectives indicating comparison (suffixes*: -er, -ier, -est, -iest).*
- Students will use oral language to discuss and debate using cooperative grouping.
- Students will justify and reason verbally and in writing.
- Students will read and write using sentence frame structure.
- Students will develop mathematical vocabulary using visual representations (charts, graphic organizers).

Materials:

- Calculator
- Deck of cards for each small group
- Dice
- Handout #1: Comparing Ratios
- Handout #2: How Does Your Hand Compare?
- Handout #3: Roll the Die and Apply
- Handout #4: Exit Slip

Common Core Learning Standards:

7.RP.1 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics

MP.6 Attend to precision.

<table>
<tr><th>Common Issues</th><th>Suggested Questions and Prompts</th></tr>
<tr><td>When representing a ratio as a percent, the student may divide incorrectly because they confuse the dividend and divisor.</td><td>Remind students that the number in the numerator is the dividend, and the denominator is the divisor.
Connect the idea of part over whole in a percent, to that of the ratio.
If the ratio of boys to girls in a class is 2:3, then what is the ratio of boys to the whole class (boys and girls)? Which would be the denominator, and which the numerator?</td></tr>
<tr><td>Converting a part-to-part ratio to a part-to-whole ratio</td><td>Are the units in the ratio two pieces, or is it a piece of a whole compared to the whole?
When a ratio is stated in the format A to B, then it is part-to-part. When it is stated as A out of B, it is part-to-whole.</td></tr>
<tr><td>Incorrectly setting up a proportion due to not considering the units</td><td>Introduce a template in the beginning, explaining why it is set up in such a manner. Then begin fading it out as students demonstrate more understanding.
Template 1:
<table>
<tr><td></td><td>Ratio A</td><td rowspan="3">=</td><td>Ratio B</td></tr>
<tr><td>Unit X</td><td></td><td></td></tr>
<tr><td>Unit Y</td><td></td><td></td></tr>
</table>
Template 2:
<table>
<tr><td></td><td>Unit X</td><td rowspan="3">=</td><td>Unit Y</td></tr>
<tr><td>Ratio A</td><td></td><td></td></tr>
<tr><td>Ratio B</td><td></td><td></td></tr>
</table></td></tr>
</table>

Vocabulary Instruction:

It is recommended to infuse new concepts and vocabulary into the lesson within a context that builds upon background knowledge for ELL students. Giving students the opportunity to read, listen, speak, and write collaboratively builds understanding about academic concepts. The activities given in this lesson provide ELL students with multiple opportunities to demonstrate understanding. A combination of incorporating real-life context and problems to demonstrate the concept application into the lesson, activities for skill practice, and visual aids—including pictures, diagrams, charts, and sentence frames—make both the language and content more accessible to students.

Focus Vocabulary Definitions

- **ratio** – a multiplicative comparison between two quantities. It can be represented as *a:b*, *a to b*, or *a/b*.
- **unit** – a quantity used as a standard of measurement.
- **rate** – the ratio of two different measurements with different units
- **unit rate** – a ratio that indicates how many units of one quantity there are for every one unit of the second quantity (e.g., miles per hour, price per pound, students per class, heartbeats per minute)

Tiered Vocabulary

Tier I	Tier II	Tier III
cellphone deck (of cards) online dealer	comparison clockwise statistic unit quarter (1/4)	ratio proportion percent unit rate

Suggested Lesson Outline (with Embedded Support for ELLs):

- Introduction to comparing ratios – Scenario #1: Strategy share-out, and discussion
- Think–pair–share: Do more teenage boys or girls use their cellphones to get online?
- Individual work: How do we compare? Strategy application, share-out, and discussion
- Practice comparing ratios:
    - Group Work – Activity #1: How Does Your Hand Compare?
    - Group Work – Activity #2: Roll the Die and Apply
- Closure
- Exit slip

Prerequisite Skills

(Use the prerequisite skills and techniques as a "do now," entrance slip, or the prior lesson's exit slip to assess students' readiness for this lesson.)

Prerequisite Skills	Techniques
Converting a fraction to	• Set up a part-to-whole proportion, and solve for the missing

<table>
<tr><td>a percent</td><td>value.
• Divide numerator (part) by denominator (whole), then multiply by 100:
3/5 = 60%
• Manipulate a part-to-whole ratio table that leads to the whole of 100:
<table>
<tr><td>Part</td><td>3</td><td>6</td><td>12</td><td>60</td></tr>
<tr><td>Whole</td><td>5</td><td>10</td><td>20</td><td>100</td></tr>
</table></td></tr>
<tr><td>Finding the unit rate</td><td>• Applying the strategies introduced in Lesson 1, divide or set up a proportion.
• Remind students of real-life applications of unit rate, such as $2.50 a pound, $3.97 per gallon, 55 miles per hour, and elicit what they have in common
• Give students two ratios, one as a unit rate and one not, and have them identify which is a unit rate and explain why.</td></tr>
<tr><td>Solving proportions</td><td>• Emphasize the importance of setting up the proportion appropriately. Have students fill in the templates
Template 1:
<table>
<tr><td></td><td>Ratio A</td><td rowspan="3">=</td><td>Ratio B</td></tr>
<tr><td>Unit X</td><td></td><td></td></tr>
<tr><td>Unit Y</td><td></td><td></td></tr>
</table>
Template 2:
<table>
<tr><td></td><td>Unit X</td><td rowspan="3">=</td><td>Unit Y</td></tr>
<tr><td>Ratio A</td><td></td><td></td></tr>
<tr><td>Ratio B</td><td></td><td></td></tr>
</table>
• Refer to charts of strategies to solve proportions from Lesson 2</td></tr>
<tr><td>Using ratio tables</td><td>• Have students fill in the missing values of a ratio table, and to indicate the operation used. Then, demonstrate how each ratio created is equivalent.</td></tr>
</table>

Lesson Description:

Real-world contexts make the material more accessible to ELL students.

Distribute ***Handout #1: Comparing Ratios***, and ask a student to read the following scenario aloud:

> Scenario: Approximately 7 out of 18 teenage girls with cellphones use their cellphones to get online. On the other hand, approximately 1 out of 4 boys with cellphones use their cellphones to get online.

Ask students to record the two ratios on the handout in the spaces provided, and to write down what each of the numbers mean.

Then, ask students:

- *Are these part-to-part or part-to-whole ratios?* [Part-to-part]
- *What are some strategies we can use to determine if more teenage girls or boys use their cellphone to get online?*

A routine of giving wait time gives ELLs the opportunity to organize their thoughts and prepare to share.

Allow for a minute wait time before having students share. Record their answers on the board or on chart paper.

The strategies that should be elicited from the students, or introduced or retaught are:

Explicit teaching benefits ELLs in providing clear and easy procedures to follow. Through this technique, ELLs learn a new skill as well as the correct

- Unit rate: Find the unit rate of each ratio.
- Common unit: Manipulate one or both of the ratios so that they have one numerical unit in common (using ratio tables).
- Ratios as percents: First, the ratio has to be represented as a part to whole, and then that part-to-whole ratio can be converted into a percent.

Students take notes of three strategies that are shared. Then, ask them to write the name of the strategy and its description in the space provided on Handout #1.

A written visual is a representation for ELLs to use as reference for speaking and writing. It's also a tool to maximize interaction as well as to build a print-rich environment.

Use these strategies to answer the following:

- In a bag of M&M's, the ratio of red candy pieces to yellow candy pieces is 4 to 12. In a bag of Skittles, the ratio of red candy pieces to yellow candy pieces is 2 to 5.

  Assuming that both bags contain the same number of pieces, which bag has more yellow pieces?

<table>
<tr><th>Strategy</th><th>M&M's</th><th>Skittles</th></tr>
<tr><td>Unit Rate</td><td>$$\frac{12\ yellows}{4\ reds} = \frac{?}{1\ red}$$
After dividing the numerator and denominator by 4, you learn that there are 3 yellows for each red.</td><td>$$\frac{5\ yellows}{2\ reds} = \frac{?}{1\ red}$$
After dividing the numerator and denominator by 2, you learn that there are 2.5 yellows for each red.</td></tr>
<tr><td rowspan="2">Common Units</td><td><table><tr><th>Red</th><th>Yellow</th></tr><tr><td>4</td><td>12</td></tr><tr><td></td><td></td></tr></table></td><td><table><tr><th>Red</th><th>Yellow</th></tr><tr><td>2</td><td>5</td></tr><tr><td>4</td><td>10</td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr><tr><td></td><td></td></tr></table>By doubling the amount of red, you now have the same number of red in each bag, and can compare the number of yellow.</td></tr>
<tr><td><table><tr><th>Red</th><th>Yellow</th></tr><tr><td>4</td><td>12</td></tr><tr><td>20</td><td>60</td></tr><tr><td></td><td></td></tr></table>Multiply the red and yellow by 5.</td><td><table><tr><th>Red</th><th>Yellow</th></tr><tr><td>2</td><td>5</td></tr><tr><td>4</td><td>10</td></tr><tr><td>24</td><td>60</td></tr><tr><td></td><td></td></tr></table>First the red and yellow are doubled, and then multiply each by 6 to arrive at the same number of yellow. Now you can compare the red.</td></tr>
<tr><td>Ratios as Percents</td><td>4 red to 12 yellow (part-to-part)
12 yellow to 16 red & yellow (part-to-whole)
$12 \div 16 = .75 = 75\%$</td><td>2 red to 5 yellow (part-to-part)
5 yellow to 7 red & yellow (part-to-whole)
$5 \div 7 = .714 = 75\%$
Once they are both represented as percent's, you can compare the percentages.</td></tr>
</table>

Before moving onto the next question, the above three strategies should be charted and posted in the classroom.

Cooperative learning supports oral development for ELLs. These techniques allow students to use different language modalities such as listening and speaking.

After the strategy share-out, tell the students that they are being given four minutes to work in pairs to answer the question that is on the handout*:*

- *Do more teenage girls or boys use their cellphones to get online?*

Tell them to show their work and to clearly write their answer and justification.

While the pairs are working, monitor their work and take note of students' strengths and misunderstandings.

Monitoring students working in groups or independent serves as a tool for teachers to make modifications and give additional instructional support.

After the four minutes are up, facilitate a class share-out where different strategically-chosen pairs of students share their work. Students have to decide whether they agree or disagree with each pair's strategy and conclusion using a thumbs-up or thumbs-down class response. For each shared strategy, ask a student with thumbs up to explain why he/she agrees, and ask a student with thumbs down to explain why he/she doesn't agree. (Choose a pair that arrives at the correct solution with an incorrect strategy, and have a student correct their reasoning.)

Next explain that the class is going to move into the second part of the lesson: How do we compare our class data? Collect data regarding cellphones from the class and record the answers on the board, while the students record the data in Part II of Handout #1. Remind students to refer to the first part of the lesson to remember which numbers belong in the numerator and denominator.

Give the following prompts to students, making sure they have enough time to write down the number of responses to each prompt:

- *If you are a girl with a cellphone, raise your hand.*
- *Raise your hand if you are a girl who uses her phone to get online.*
- *If you are a boy with a cellphone, raise your hand*
- *Raise your hand if you are a boy who uses his phone to get online.*

On the handout, have students record the number of girls who have a cellphone, and how many of them use their cellphone to get online. They will do the same for the boys.

Then, ask students to work individually to decide if the class ratios are greater or lesser than the statistical ratios. Have them show their work and solutions on the handout.

Assign students to small groups, and have each group member share his/her answer and explanation. While one student in the group presents, the others listen and either agree with the presenter's work and solution or help guide the presenter into making corrections in reasoning. When the small groups have completed sharing among themselves, regroup the class and ask one person from each group to share their findings with the class.

Next, use the following activities to enhance students' understanding of how to compare ratios, and to allow students to practice applying the strategies introduced in class.

Sufficient practice gives ELLs opportunities to master the new content taught as well as becoming proficient in the focused modality.

Begin by assigning students to work in group of four to five, and distributing ***Handout #2:*** Review the directions for Part I with the class, and demonstrate how the handout should be filled in by writing an example on the board.

Note to Teacher: The directions state that the student with the highest number for a birthday (e.g., 22, if your birthday is April 22) is the dealer for Round 1. Tell the class that if two or more students have the same numbered birthday, the student with the highest number for a month is the dealer.

As students work on the activity, circulate around the room, making sure they are correctly following the directions.

When the activity is complete, have students answer the reflection questions in Part II of the handout. Then, call on students to share their answers with the class, making sure to correct any misconceptions they might have. *It doesn't make sense to have students work on Handout 2, then go back to Handout 1. Therefore, I moved Part III of Handout 1, making it Part II of Handout 2.*

For the next activity, assign students to groups of four. Distribute ***Handout #3: Roll the Die and Apply*** and one die to each group. (You might want to explain that die is the singular of dice, a word students will probably be more familiar with.) Review the directions for this activity with class, and demonstrate how the handout should be filled in by writing an example on the board.

Allow students sufficient time to solve all six problems on the handout. As each problem is solved, group members individually share their work with the others to if the strategy for each problem was applied correctly. The group must agree on the solution to each problem before moving on to the.

As students work on the activity, circulate around the room, making sure they are correctly following the directions. When they completed the activity, call on students from each group to share their findings with the class.

Closure:

Referring to the charted strategies and student work created throughout the lesson, lead a class discussion. Be sure to ask students the following questions:

- *Which strategy for comparing ratios was the most useful? Why?*
- *Which strategy for comparing ratios has the most limitations? Why?*
- *What do you think will be an error that someone might make when comparing ratios?*
- *Why is important to recognize and differentiate between part-to-part and part-to-whole ratios?*

Assessment:

Distribute ***Handout #4: Exit Slip*** to students and tell them to answer the solve the problems individually. When time is up, collect the handouts, making sure that students' name appear on them. Use the exit slips to assess students' comprehension of the lesson.

Lesson 3: Do More Girls or Boys Use Their Cellphones to Get Online?

Handout #1: Comparing Ratios

Name: _____ Date: _____

<u>Part I</u>

<u>Scenario</u>: Approximately 7 out of 18 teenage girls with cellphones use their cellphones to get online. On the other hand, approximately 1 out of 4 boys with cellphones use them to get online.

Write the ratios mentioned in the scenario in fraction form, and the meaning of each of the numbers.

Girls Ratio	Meaning of the Numbers
$\frac{\square}{\square}$	➔ _____ ➔

Boys Ratio	Meaning of the Numbers
$\frac{\square}{\square}$	➔ _____ ➔

Strategies used to compare ratios:

Name of Strategy	**Description of Strategy**

<u>Question</u>:

Do more teenage boys or girls use their cellphones to get online? Show your work in the space below, and clearly write your answer and reasoning.

Work Space:

Answer and Reasoning:

_____

_____

_____

_____

_____

Part II

How do we compare our class data?

Our class data:

Our Class Girls Ratio	Our Class Boys Ratio
$\frac{\square}{\square}$	$\frac{\square}{\square}$

Are our class ratios greater or less then the statistics' ratios? Explain.

Work Space:

Answer and Reasoning:

_____

_____

_____

_____

_____

Lesson 3: Do More Girls or Boys Use Their Cellphones to Get Online?

Handout #2: How Does Your Hand Compare?

Name: _____ Date: _____

Part I

Directions: Students work in groups of 4 to 5. The student with the highest number for a birthday (e.g., 22, if your birthday is April 22) is the dealer for Round 1. (If two or more students have the same numbered birthday, the student with the highest number for a month is the dealer.) After that, the role of dealer is assigned on a clockwise rotation for Rounds 2 through 5. The dealer begins by dealing to the person on his/her left, and then continues to deal clockwise around the table. Each student tells the dealer they number of cards he/she wants. After dealing cards to everyone else, the dealer deals the number of cards he/she wants to himself/herself.

Once the cards are dealt, each student shares his/her red-to-whole, black-to-whole, and red-to-black ratios, and all group members record that information in the spaces provided on the chart below. After all ratios for each round are recorded, the student with the highest ratio, per category, gets that ratio circled and earns 20 points that round.

		Player 1	Player 2	Player 3	Player 4	Player 5
Round 1	Red:Whole					
	Black:Whole					
	Red:Black					
Round 2	Red:Whole					
	Black:Whole					
	Red:Black					
Round 3	Red: Whole					
	Black:Whole					
	Red:Black					
Round 4	Red: Whole					
	Black:Whole					
	Red:Black					

Round 5	Red:Whole					
	Black:Whole					
	Red:Black					
Total Points						

Part II

Directions: After completing the activity in Part I, answer the following questions:

1. How did the number of cards you asked for affect your ratios?

   _____

   _____

   _____

   _____

2. In what situation could one of the ratios be greater than one?

   _____

   _____

   _____

   _____

3. In Round 1, Kate had fewer cards than in Round 2. In Round 2, she had fewer red cards than in Round 1. In which round did she have more red cards?

   Explain your answer:

   _____

   _____

   _____

   _____

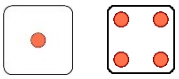
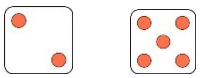
Lesson 3: Do More Girls or Boys Use Their Cellphones to Go Online?

Handout #3: Roll the Die and Apply

Name: _____ **Date:** _____

Directions: Students work in groups of four. For each of the six problems given below, each student rolls the die to determine which strategy to apply. If the strategy does not apply to the problem, the student can roll the die again. Then, each student works independently to solve the problem using that strategy. When all members of the group have finished, each member shares his/her work and solution. While sharing, group members check to see if the strategy is applied correctly. The group has to agree on a solution before moving on. In their explanations, students should use at least one of these vocabulary words: *ratio, unit rate, proportion, unit.*

If you roll a:

 Use the unit rate strategy. 1 or 4	 Use the common unit strategy. 2 or 5
Use the ratios-as-percents strategy. 3	Free choice – Use any appropriate strategy. 6

Problem 1: During a basketball game, Thomas makes 8 of 11 free throws. Francis makes 10 of 13 free throws. Who is better at making free throws? ***Strategy:*** _____

Work Space	*Explanation*

Problem 2: Jason and Wilson are the pitchers in a baseball game. Their coach is keeping count of the pitches they throw. The coach told Jason that his ratio of fast balls to curveballs is 7 to 6. The coach told Wilson that his ratio of fast balls to curveballs is 9 to 8. Who pitched a greater ratio of fast balls, Jason or Wilson? ***Strategy:*** _____

Work Space	*Explanation*

Problem 3: Rita takes nine steps in five seconds. Gavin takes 12 steps in seven seconds. Who takes more steps per second? ***Strategy:*** _____

Work Space	*Explanation*

Problem 4: Ryan bought two and a half pounds of American cheese for $15.60. Tomas bought a quarter pound of Swiss cheese for $1.41. Is the American or Swiss cheese less expensive? ***Strategy:*** _____

Work Space	*Explanation*

Problem 5: Louis went to the Mighty gas station and it took him 7.2 minutes to get 31.464 gallons of gasoline. The following week, he went to AM Gas and it took him six and a half minutes to get 34.775 gallons of gasoline. At which gas station, Mighty or AM Gas, did the gasoline pump faster? ***Strategy:*** _____

Work Space	*Explanation*

Problem: Carmine is choosing flooring for his new house. Wood flooring cost $81.50 for a 5-foot by 5-foot area. Carpeting costs $74.40 for an 8-foot by 3-foot area. Ceramic tiles cost $146.16 for a box of 42 tiles that each measure 1 foot by 1 foot. List the cost of the flooring from the least to the greatest. ***Strategy:*** _____ Originally called "Challenge," not Problem.

Work Space	*Explanation*

Lesson 3: Do More Girls or Boys Use Their Cellphones to Get Online?

Handout #4: Exit Slip

Name: _____ Date: _____

Word Bank

ratio	***unit rate***	***proportion***

1. Write and solve a word problem that uses at least one of the vocabulary words from the word bank. The problem must be solved by using ratios or proportions.

   Write the problem:

   _____

   _____

   _____

   Solve the problem:

2. Look at the following ratios:

   $\frac{13}{18}$ 8 to 11 15:21

   Now put them in order, from least to greatest, and justify your answer:

   From least to greatest:

   Justification:

   _____

   _____

   _____

   _____

3. On the last unit test, there were 27 questions. Sara correctly answered 23 questions. On this unit test, there are 31 questions. Sara thinks that if she correctly answers 26 questions, she will have done better than on the last unit test. Is she correct? Answer “Yes” or “No” and then explain your answer.

   Work space:

   Is she correct? _____

   Explanation:

   _____

   _____

   _____

   _____

   _____

Lesson 4: Qualitative Comparisons and Proportionality Using a Graph

Mathematical Goals:

- Students will understand qualitative predictions and comparisons using proportional reasoning.
- Students will read and interpret linear graphs (Quadrant I).

Language Goals:

- Students will use oral language to discuss and debate using cooperative grouping.
- Students will justify and reason verbally and in writing.
- Students will read and write using sentence frame structure.

Materials:

- Color counters
- Graph paper
- Handout #1: Entry Slip
- Handout #2: Scenario 1
- Handout #3: Scenarios 2 & 3
- Handout #4: Scenarios 4 & 5
- Handout #5: Extension Problems
- Handout #6: Exit Slip

Common Core Learning Standards

7.RP.2 Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.4 Model with mathematics

MP.6 Attend to precision.

Common Issues	Suggested Questions and Prompts
Student confuses the *x* and the *y* coordinates.	Explain to student that points are organized as ordered pairs. Ask the student which coordinate goes first and which goes second in an ordered pair.
Student confuses the *x*- and the *y*-axis.	Have a diagram of the *x*-*y* plane in the classroom that students can refer to.
Student counts points instead of intervals.	Ask whether distances are the same as points: • *Are you counting the points or the distances between points?*
Student incorrectly believes that the *y*-intercept is just the *y*-coordinate. For example, student says the *y* intercept is *b,* not the point (*0,b*).	Remind students that every point on the Cartesian plane has an *x*- and a *y*-coordinate. Then, ask: • *If b is the y-coordinate for the y-intercept, what is the x-coordinate value for a y-intercept?*

Vocabulary Instruction:

Vocabulary Instruction

Throughout this unit, we have recommended explicit vocabulary instruction by teaching important words and reinforcing them. In this lesson we activate prior knowledge, and use graphs and collaborative group work to promote oral language development. We provide students with opportunity to explain their math thinking in writing in order to support written language development

Focus Vocabulary Definitions

- ***y*-intercept** – a point of the graph that intersects the *y*-axis.
- **origin** (on a graph) – the point where the *x*- and *y*-axes intersect.
- **ratio** – a multiplicative comparison between two quantities. It can be represented as *a:b, a to b*, or $\frac{a}{b}$.
- **proportional relationship** – a relationship between two quantities in which the two quantities vary directly with each other.

Tiered Vocabulary

Tier I	Tier II	Tier III
hours	liters	non-proportional relationship
	Minutes	proportional relationship
	miles	Coordinates

Suggested Lesson Outline (with Embedded Support for ELLs):

Prerequisite Skills:

(Use the prerequisite skills and techniques as a "do now," entrance slip, or the prior lesson's exit slip to assess students' readiness for this lesson.)

Prerequisite Skills	Sample Questions
Define and identify a ratio.	• Have the definition charted and visible for students to be able to refer back to. • Have a pictorial representation with the definition to make connections. • Show the different ways to represent a ratio (e.g., 2 to 5; 2:5; $\frac{2}{5}$).
Understand equivalent fractions.	• Have fraction strips available (cut-outs). • Fraction manipulative.
Familiarity with the *x-y* coordinate plane.	• Have posters of the *x-y* plane posted in the classroom for student reference .

Lesson Description

As students come in, have them work independently for 8-10 minutes on the ***Handout #1: Entry Slip***.

After you have collected the completed entry slips from students (making sure students' names appear on them), arrange the students in pairs and distribute ***Handout #2: Scenario 1***

Have students use the think-pair-share strategy for this activity. Students read the scenario and think about problems posed individually. They then work together with their partner to solve the problems and, if called on, share their responses with the class. Additionally, it is suggested that you have color counters available for students who may request them for their work on this activity.

In order to facilitate a discussion, ask the following questions:

- *Can you rephrase the question in your own words?*
- *What does it mean to have a 3 to 5 ratio in this scenario?*
- *Does it matter how many red and yellow counters there are?*
- *What is the minimum number of red and yellow counters that can be added?*
- *What happens to the ratio as you add the same number of red and yellow counters? Does it change? If so, how?*

Select a pair of students with the right answers and an adequate justification, and have them present and explain their work for the entire class. Keep in mind that the purpose of this activity is to disabuse students of the notion that "if you do the same thing to the numerator and denominator of a fraction representing a ratio, then it will not change."

After completing the activity on Handout #2, transition to discussing proportionality and non-proportionality using ***Handout #3: Scenarios 2 & 3***. Distribute the handout, which includes two graphs (A and B), to students, who continue to work in pairs.

In the initial part of the activity on Handout #3, students will be asked to match each scenario to one of the graphs. Students should be given five minutes to work with their pair partner and match the scenarios to the graphs. As the students work on this activity, circulate around the classroom guiding students and assisting them when needed. In order to facilitate a discussion, you might ask the following questions:

- *What does the x-axis represent?*
- *What does the y-axis represent?*
- *What is the name of the point at which the graph intersects the y-axis?*
- *Is it important that one of the buckets is empty and the other is not?*
- *One of the buckets has two liters before water is added to it. How is this reflected on the graph?*

Building vocabulary from the concepts. The y-intercept is simply a label for the point where the graph intersects the *y*-axis. It is more important for students to first understand a concept and then attach a label to it.

After the class discussion and the successful matching of the graphs with the two scenarios, tell students:

> *Now that we know that Scenario 3 is the verbal representation of the graph that intersects the y-axis at the point (0,2), because the bucket in Scenario 3 has 2 liters of water to begin with (time 0 minutes).*

Write the following on the board and read it aloud:

> *What are some of the ways you know to determine whether a relationship is proportional?*

Sample student answers may include using a ratio table to see whether the ratio stays constant, and setting a proportion between two ratios and seeing whether they are equal.

To help activate the students' prior knowledge, ask them to recall problems and examples from previous lessons on ratios and proportionality. Be mindful to give them adequate time to recall the information and to formulate an answer.

> Wait time gives ELLs the chance to understand what's being asked, and to formulate a cogent answer.

After eliciting responses from students, explain that when we look a graph—which should be displayed prominently—some characteristics that allow us to recognize it as a proportional relationship are:

1. It will always be a straight line.
2. It will always go through the origin (0,0) on a graph.
3. It will always have a constant rate of change.

> Writing a concise version of the important concepts in an activity will help ELLs by having that information represented in multiple ways.

Next, have students work individually with Graph B, using the above criteria (1–3) to establish whether it represents a proportional relationship. Circulate around the classroom looking at students' work, ensuring that they are checking whether Graph B is a straight line (it is), the ratio $\frac{y}{x}$ is constant (it is not), and whether it goes through the origin (it does not).

Ask a student to write on the board that Graph B does not represent a proportional relationship, even though it is a straight line, since it does not go through the origin (0,0), and it does not have a constant $\frac{y}{x}$.

Distribute ***Handout #4: Scenarios 4 & 5***, which includes two graphs (C and D), and instruct students to work independently on the two scenarios. Circulate around the room, checking students' understanding of the activity. It is suggested that you take note of (and quickly write down) how many students are completing the handout with ease and which ones are struggling. If you start to notice the same error by four or more students, you may want to call the attention of the entire class to this issue and go over it.

When time is up, call on some students to share their work with the class. Engage the students in discussion and correct any errors or misconceptions they might have.

Next, distribute ***Handout #5: Extension Problems***, which has additional challenging problems from previous lessons on proportionality.

Closure

Lead a whole-class discussion. Refer to the charted criteria for proportionality and classwork to answer the following questions:

- Why did the proportional and non-proportional relationships that we discussed in class both have graphs that were straight lines?
- If we are given the graph of a straight line, what is one way we can tell whether it represents a proportional relationship?

Assessment

Distribute ***Handout #6: Exit Slip***, which comprises three end-of-lesson questions for students to answer independently. Give students eight minutes to answer the questions. Then, collect the handouts, making sure students' names appear on them. Use the exit slips to assess students' comprehension of the lesson.

Lesson 4: Qualitative Comparisons and Proportioning Using a Graph

Handout #1: Entry Slip

Name: _____ Date: _____

Word Problem:

If India has more cellphones than Italy, how is it possible that Italy has the higher ratio of cellphone owners?

1. Describe how this is possible:

_____

_____

_____

_____

_____

_____

2. Explain your reasoning:

_____

_____

_____

_____

_____

Lesson 4: Qualitative Comparisons and Proportionality Using a Graph

Handout #2: Scenario 1

Name: _____ Date: _____

Directions:

- Read Scenario 1.
- Working with a partner, use the think-pair-share strategy to respond to the prompts that follow.
- Be ready to share your responses with the whole class.

Scenario 1

The ratio of red counters to yellow counters placed in a paper bag is 3:5. Then, more counters are added to the bag—the same number of each color. Maria states that the ratio is preserved since the same number of yellow and red counters was added to the bag. Is she correct? Justify your answer.

1. Do you agree or disagree with Maria?

   _____

2. Explain why you agree or disagree with Maria:

   _____

   _____

   _____

   _____

   _____

Lesson 4: Qualitative Comparisons and Proportionality Using a Graph

Handout #3: Scenarios 2 & 3 (page 1 of 3)

Name: _____ Date: _____

Directions:

- Read the following scenarios and match them with the corresponding graphs (A and B) that follow.
- Be ready to share your answer and justification with the class.

Scenario 2

A hose is filling an empty bucket of water at a constant rate of 3 liters per minute.

This scenario matches Graph ____.

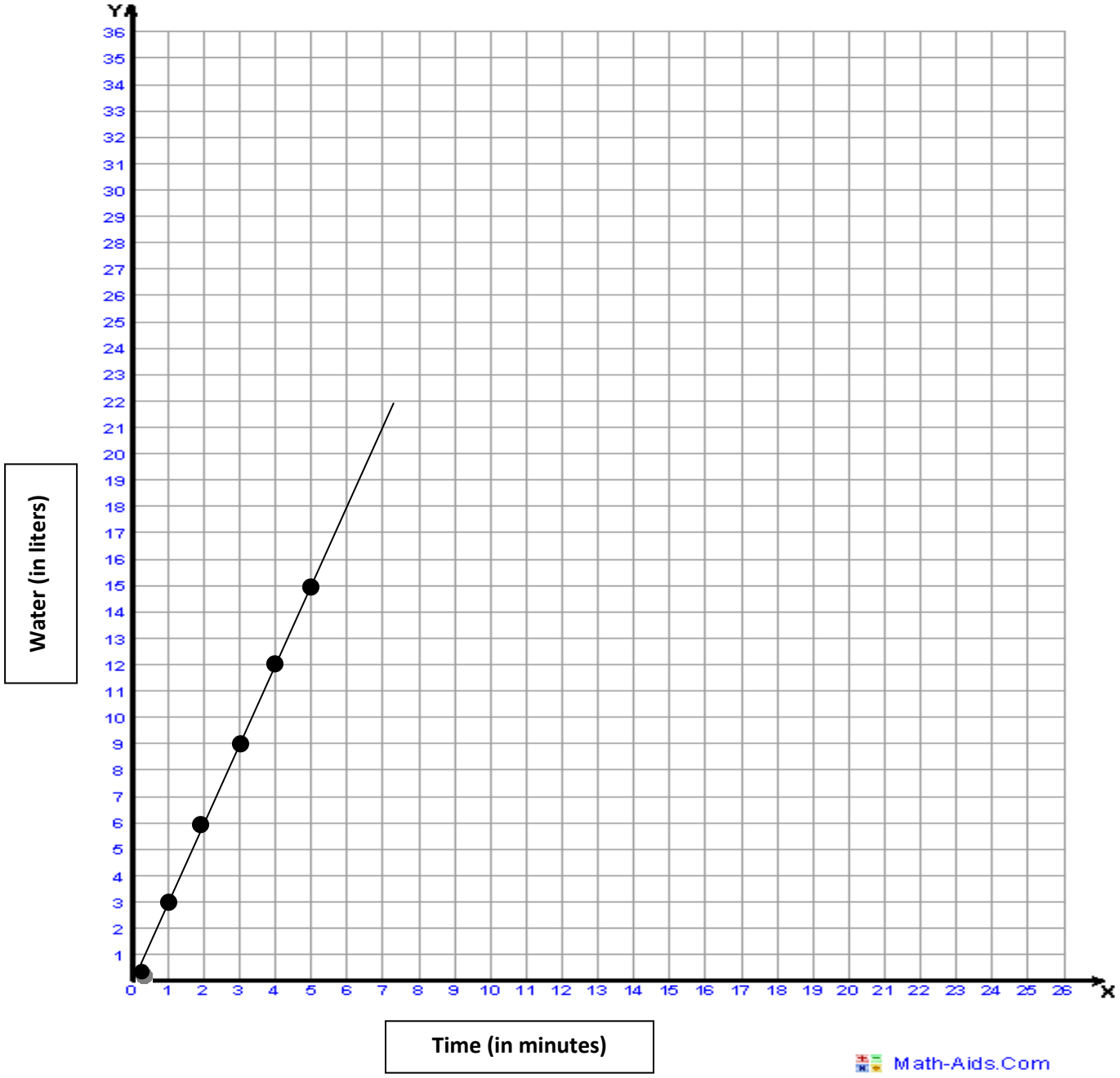
Scenario 3

A bucket has 2 liters of water in it. It is being filled at a constant rate of 3 liters per minute.

This scenario matches Graph ____.

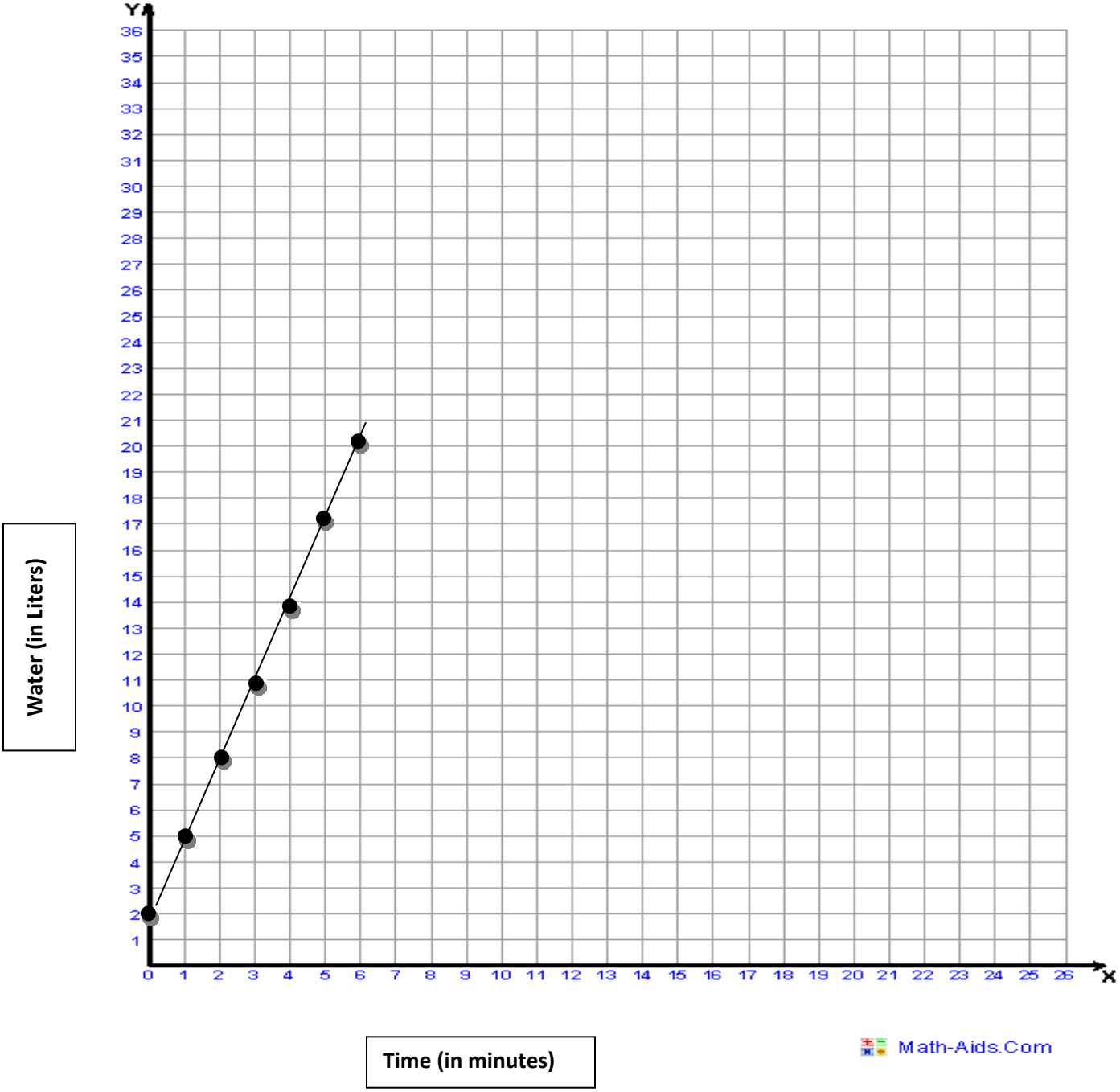
Handout #3: Scenarios 2 & 3 (page 2 of 3)

Graph A



Handout #3: Scenarios 2 & 3 (page 3 of 3)

Graph B



Lesson 4: Qualitative Comparisons and Proportionality Using a Graph

Handout #4: Scenarios 4 & 5 (page 1 of 3)

Name: _____ Date: _____

Directions:

- Read the following scenarios and match them with the corresponding graphs (C or D) that follow.
- Be ready to share your answers and justification with the class.

Scenario 4

Fast Wheels Taxi Company charges an initial fee of $3 plus $1 dollar for every mile traveled.

This scenario matches Graph ____.

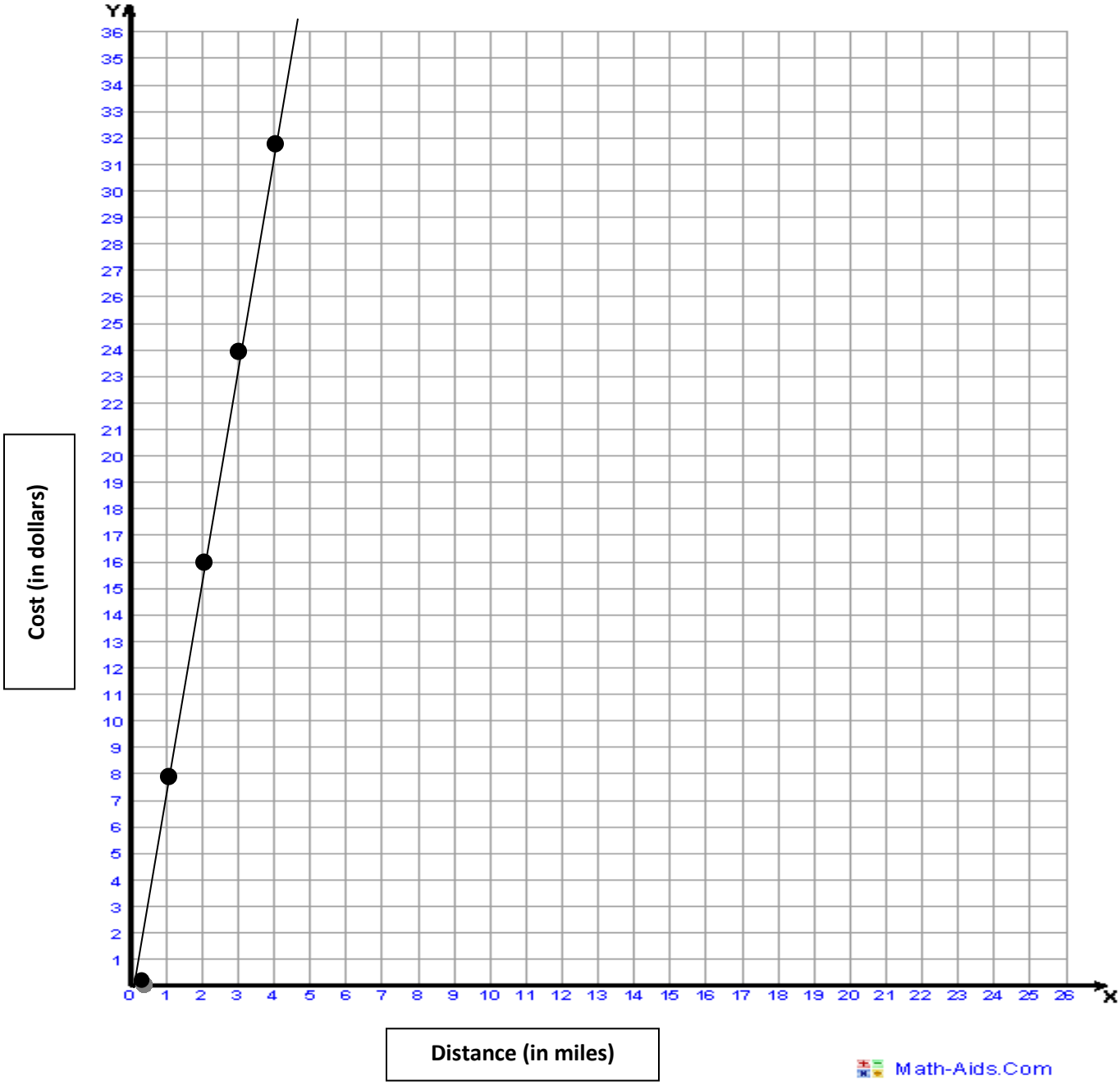
Scenario 5

Luxury Limousines Company charges $8 for every mile and no initial fee.

This scenario matches Graph ____.

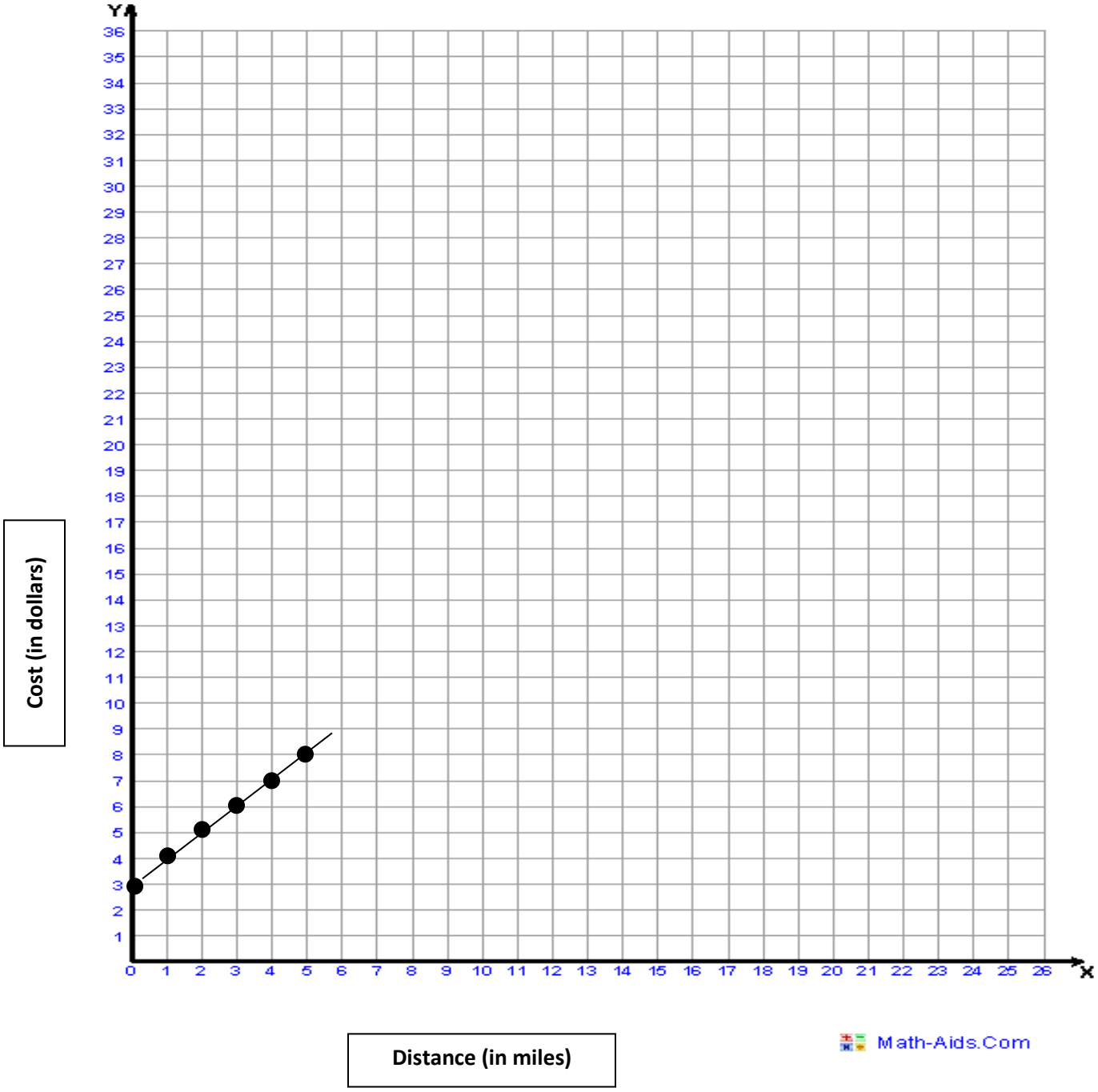
Handout #4: Scenarios 2 & 3 (page 2 of 3)

Graph C



Handout #4: Scenarios 2 & 3 (page 3 of 3)

Graph D



Lesson 4: Qualitative Comparisons and Proportionality Using a Graph

Handout #5: Extension Problems

Name: _____ **Date:** _____

Directions: Answer the following questions in the space provided.

1.) The ratio of green to red marbles in a jar is 3:7. Which column in the ratio table below is incorrect? (Circle your answer and justify it below).

Green marbles	9	51	6	60	45	48	36
Red marbles	21	119	14	140	100	112	84

Justification:

_____

_____

_____

_____

_____

2.) In a 7th grade class, the ratio of girls to boys is 3:5. Which of the following could not be the total number of boys and girls in the class? (Circle your answer and justify it below.)

a) 24 b) 40 c) 35 d) 32

Justification:

_____

_____

_____

3.) In the problem above, if the 3:5 ratio were of pounds of sugar to pounds of salt (instead of girls to boys), could 35 have been the total number of pounds of sugar and salt? (Check either “Yes” or “No” and justify your answer below.)

Yes ____ No ____

Justification:

_____

_____

_____

_____

_____

4.) A class of students is split into two groups and sent to different rooms. They are asked to stand in line. The larger number of students is sent to Room A. The length of Room A is shorter than the length of Room B, but the widths of both rooms are the same. If the students are all standing in a straight line, in which of the rooms are they closer to each other? (Check either “Room A” or “Room B” and justify your answer below.)

Room A _____ Room B _____

Justification:

_____

_____

_____

_____

_____

Lesson 4: Qualitative Comparisons and Proportionality

Handout #6: Exit Slip

Name: _____ Date: _____

Directions: Answer the following questions in the spaces provided.

1) Today Juan is using more iced tea powder and less water than he used for his iced tea two days ago. How will today's iced tea taste? (Circle your answer.)

   a) Stronger than two days ago

   b) Weaker

   c) The same

2) How can we tell whether a relationship is proportional by looking at its graph?

   _____

   _____

   _____

   _____

   _____

3) A linear graph always represents a proportional relationship. (Check "True" or "False" and justify your answer below.)

   True _____ False _____

   Justification:

   _____

   _____

   _____

   _____

   _____

Lesson 3: The Cost of Texting

Mathematical Goals:

- Students will translate from a ratio table to a pair of numbers in the coordinate plane (Quadrant I only) to a linear graph, understanding how these are the same representations of a proportional relationship.
- Students will represent proportional relationships as an equation of the form: $y = mx$, where m is the proportionality constant.

Language Goals:

- Students will compare and contrast proportional relationships from their graphs.
- Students will debate and discuss their findings in collaborative groups.
- Students will justify their reasoning orally and in writing.

Materials:

- Handout #1: Entry Slip (The Cost of Music)
- Handout #2: Which Is the Best Phone Plan?
- Extension for differentiation or investigation: Lesson 5 Handouts #3(1) and #3(2) (Problem-Solving Task 1: Painting the Clubhouse) (optional).
- Chart paper or any other tool useful to keep a record of students' responses.
- Tools for graphing should be available: rules, pencils with erasers, graph paper
- Calculators.

Common Core Learning Standards:

7.RP.2c. Represent proportional relationships by equations.

7.RP.2d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation, with special attention to the point (0,0) and $(1,r)$ where r is the ratio unit.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.2 Reason abstractly and quantitatively.

Common Issues	Suggested Questions and Prompts:
Graphing a ratio relationship	Refer to Lesson 4, Scenario 3, filling up a bucket. Ask: • *What information did we use to create a table?* • *How did we graph those points?*
Finding missing parts of a proportion.	Refer to Lesson 2, three strategies to find the missing part in a given proportion.

Vocabulary Instruction:

From the start of the unit, it has been recommended to do explicit instruction of vocabulary, i.e., teaching the most important words used in context, as well as providing reinforcement in content specific situations.

In this lesson we activate prior knowledge with an entry slip: Handout #1. We use visuals in the form of graphs and charts and use think-pair-share or collaborative groups to promote oral language development, and provide an opportunity to explain mathematical thinking in writing with an exit slip to foster using the vocabulary learned and complete sentences in order to provide written language development.

This lesson presents some vocabulary that teachers should address when appropriate. Potential words that might challenge ELLs are summarized in the tiered vocabulary table below.

Focus Vocabulary Definitions

- **ratio** – a multiplicative comparison between two quantities. It can be represented in three ways:
  - *a:b*
  - *a to b*
  - *a/b*
- **proportional relationship** – a relationship between two quantities in which the two quantities vary directly with one another.
- **fraction** – represents a part of a whole or, more generally, any number or equal parts.
- **product** – in mathematics, the product is the result of a multiplication.

Tiered Vocabulary

Tier I	Tier II	Tier III
Results online download discount sales plan	Research represent locate argument	Ratio proportional relationship fraction product

Suggested Lesson Outline (with Embedded Support for ELLs):

- Introduction: Handout #1: Entry Slip – The Cost of Music, think-pair-share, and class discussion
- Lesson Development: Handout #2 – Which Is the Best Phone Plan?, group work, and class discussion
- Handout #3: Exit Slip
- Extension (optional): Problem Solving Task. Painting the Clubhouse from http://illuminations.nctm.org

Prerequisite Skills:

(Use the prerequisite skills and techniques as a "do now," an entrance slip, or prior lesson's exit slip to assess student readiness for the lesson.)

Prerequisite Skills	Techniques
Understanding that "50%" and "half" both mean the same	Have students write 50% as a fraction with denominator of 100, and ask them to reduce it to lowest terms.
Locating points in the coordinate plane	Remind students that the horizontal number line represents the *x* value and the vertical line represents the *y* value on the pair (*x,y*).

Lesson Description:

Directions:

Give each student ***Handout #1: Entry Slip – The Cost of Music*** or have it displayed on the SmartBoard.
Have one student read the Scenario # 1 aloud while the rest of the class reads along silently.

Reading aloud gives ELLs a chance to participate equally, to develop a greater sense of community.

Allow five minutes for thinking before starting a class discussion. A think-pair-share strategy can be used here before the class discussion.

Think-pair-share helps ELLs to negotiate meanings of vocabulary and concepts with a classmate.

The following are questions that can be used for the whole-class discussion. Record students' responses using chart paper to validate their answers and keep all students engaged and informed:

- *What information did you use to locate the point representing the cost of two CDs at regular price?*
- *What information did you use to locate the point representing the cost of five CDs at 50% discount?*
- *Can you find a way in which the two graphs are alike? In which way are they different?*
- *Can we use what we know to find the cost of any number of CDs at either regular or sale price?*
- *What will be a number sentence to represent finding the cost of any number of CDs?*

Scaffolding questions provides ELLs with opportunities to develop linguistic fluency in spontaneous discussions.

It is most important to record the students' examples of number sentences elicited in the last question since they will be used as a point of departure to write an equation in one of the tasks that follow.

Distribute ***Handout #2: Which Is the Best Phone Plan?*** and instruct students to join their assigned groups. Ask individual students to read the problem aloud and, after allowing for clarifying questions, instruct groups to start working. Circulate around the room to monitor the interactions within each group.

Cooperative learning supports ELLs in oral language development through listening and speaking modalities.

Allow for 8 to 10 minutes of group work, at which point most students should be done with Scenario 1. Bring the class together for a discussion period with student participation. Here are some questions to facilitate the discussion:

- *Does anyone feel that either Horizon or BeMobile is a better plan?*

Mid-lesson monitoring is a tool to use as informal assessment. It provides necessary feedback to confirm whether ELL students need additional instructional support.

- *What factors seem to affect the cost of Maya's bill in a given month?*
- *In your opinion, how likely is it that Maya will send fewer than 34 texts in one month?*

The point of these mid-lesson questions is two-fold: to activate prior schema in the students and to make sure that all students have the correct answers to the questions. Therefore, it is important to also elicit answers from the students, ask them for justification of their work, and be open to questions from those students who did not get correct answers.

Ask students whether they can write an expression to summarize how they have been calculating the different costs under each payment plan. At this point, it will be useful to look at the initial student responses during the discussion of Handout #1, in particular to the question:

- *What will be a number sentence to represent finding the cost of any the number of CDs?*

We recommend that these be kept on a separate sheet of chart paper. This is a good time to use students' answers and help them to see that we can use variables (c for cost, n for the number of CDs) and the number sentence: The cost (c) of any number of CDs (n) at regular price is: $c = \$2(n)$. Subsequently, for the cost of texting, the sentence on the cost of texting using Horizon Pay-per-Text plan can be found using the equation: $c = \$0.20(n)$.

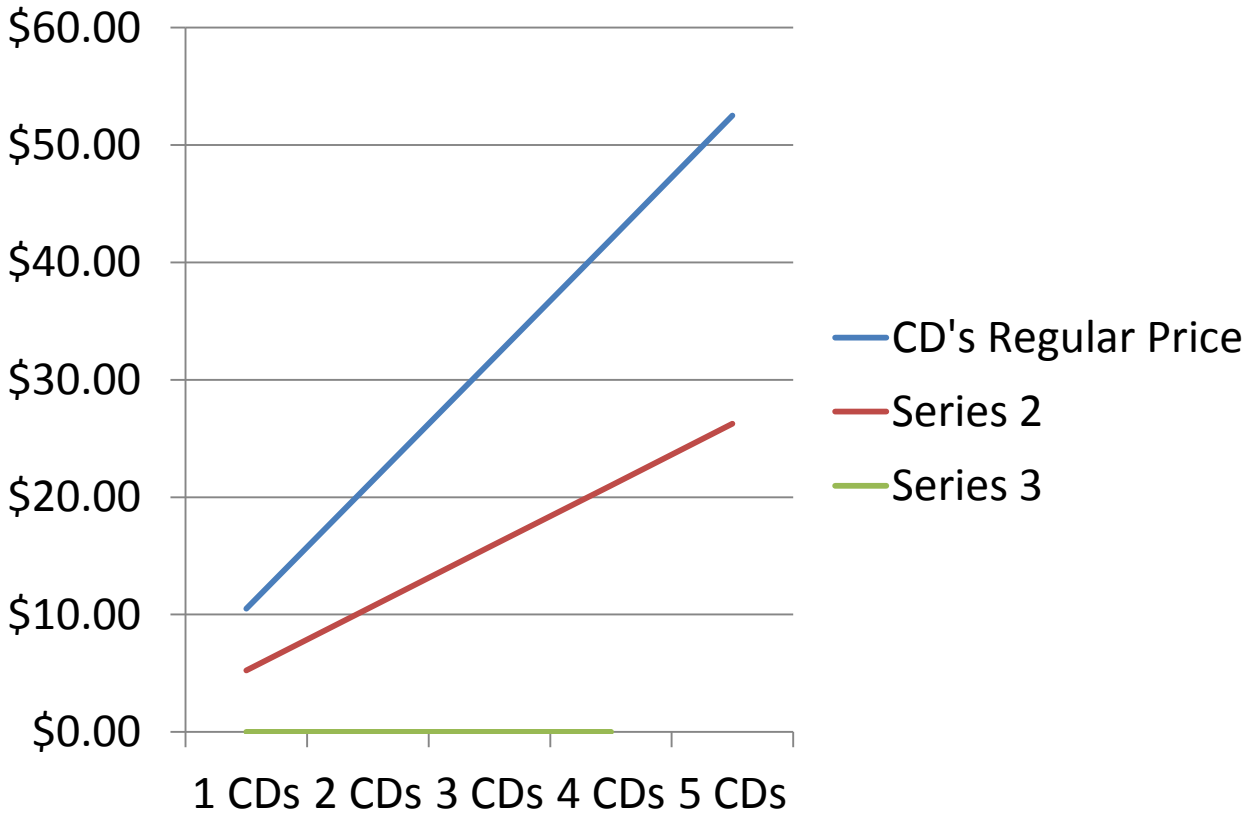
Instruct students to continue working in their groups on Scenario 2 of Handout #2: Which Is the Best Phone Plan?

Monitor time at this point. If needed, students can complete their work at home independently. As students work in groups, you have an opportunity to ask them questions and oversee their work, as well as listen to the discussions to check whether students have acquired all the concepts and skills indicated for this lesson, namely: to develop the ability to understand multiple forms of a proportional relationship, and to go back and forth among a linear representation, a coordinate pair or a ratio table, and an equation of the form $y = mx$.

Note that if the students need more time to complete Handout #2 they may not be ready to write their arguments to help Maya change phone plans.

Handout #3: Exit Slip is offered as an extension of these concepts and skills. It has been provided as optional, and it may be used with accelerated groups, or could be used as a portfolio piece, an investigation or long-term project assigned during a vacation break, for example. After distributing Handout #3, allow 5 to 10 minutes for students, working independently, to complete. When time is up, collect the exit slips, making sure that students' names appear on them so their understanding of the lesson can be assessed.

A written justification provides ELLs the opportunity to express their strengths and weaknesses, allowing the teacher to make decisions for further instruction.


$60.00
$50.00
$40.00
$30.00
$20.00
$10.00
$0.00
1 CDs
2 CDs
3 CDs
4 CDs
5 CDs
CD's Regular Price
Series 2
Series 3

Lesson 5 Handout #2

Which is the Best Phone Plan?

Instructions: 1. Read carefully until you understand what you have to do.
2. Ask clarifying question.
3. Discuss your work with your group members

Maya's parents are asking her to do less texting. She believes that the problem is the family plan they are using. In your groups you will be answering some questions that can help you to understand the situation and develop reasons that can help Maya to convince her parents that they need to change plans.

Monthly Text Plans available:

Horizon:

Pay-per-text	**Packages**
• $.20 per text • $.25 per photo text	(Includes photo texts) • 1,000 messages $10 per month • Unlimited messages $20 per month

BeMobile:

Pay-per-text	**Packages**
• $.20 per text • $.30 per photo text	(Includes photo texts) • Unlimited messages $20 per month

Scenario 1:

1. If Maya's family is using the Horizon plan and she sends 105 regular text messages in a month,

   a) What will be the cost if they are using the pay-per-text option?
   b) How much more (or less) will it cost if they use the 1,000-message plan?

Work Space

a)	b)

2. Maya sends 30 photo texts and 34 regular texts during a given month.

   a) Which company's pay-per-text plan will cost less money?

   b) How much more would the other company's plan cost?

Work Space

a)	b)

Scenario 2:

The reality is that a teenager texts a lot more than what we described in scenario 1. Research shows that teens each send 60 text messages on a daily basis. Older girls text even more: according to research, girls aged 14 to 17 send 100 texts a day. Maya is 12 years old and she sends 75 texts each day. She also sends 10 pictures each week.

2. Fill in all information missing on the table below:

Text messages	Horizon	BeMobile
Total number of texts sent by Maya in a month:	Calculate the cost of texts	Calculate the cost of texts

Work Space

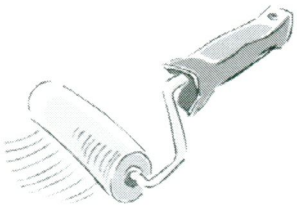
Pictures	Horizon	BeMobile
Total number of pictures sent in a month:	Calculate the cost	Calculate the cost

TOTAL COST FOR EACH PLAN:

Problem-Solving Tasks

NAME _____

Task 1: Painting the Clubhouse



STORE	PRICE
Paint Depot	$10 per gallon, no matter how many you buy
Colorful Paint Co.	$13 per gallon; if you buy 3 gallons, the 4th gallon is free
Colors R Us	pay full price of $18 for first gallon; all additional gallons are ½ price

1. Complete the table below to compare prices for 1 through 10 gallons of paint.

	PAINT DEPOT	COLORFUL PAINT CO.	COLORS R US
1 gallon			
2 gallons			
3 gallons			
4 gallons			
5 gallons			
6 gallons			
7 gallons			
8 gallons			
9 gallons			
10 gallons			

2. Where should you purchase the following amounts of paint?

3 gallons: _____

4 gallons: _____

10 gallons: _____

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3. Make a graph of the paint prices. Use different colors to plot the prices for each of the three stores.

a) Explain the pattern in your graphs.

b) Which graph shows that you can buy 4 gallons for the same price as 3 gallons?

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Lesson 5: Exit Slip

Write an argument to explain to Maya's parents why it is better to get either company's package plan. Decide which plan is best in your opinion and explain why. (Use words like unit rate, ratio, comparison or unit in your writing).
You can also use graphs, a table, an equation or any other mathematics to help Maya convince her parents. Use a separate sheet if necessary.

Name: _____

Ms. Picasso, the art teacher, has a sink in her room. The students clean their paint brushes in the sink, and this morning Ms. Picasso noticed that the sink is clogged. To make the situation worse, the faucet is dripping at a constant rate. The sink can hold 200 liters. The plumber tells Ms. Picasso that at this rate, the sink will be full in 10 hours.

A) How many liters will fill the sink per hour?
B) How many liters of water will fill the sink after 2 ½ hours? Show your work and explain your reasoning.
C) Fill in the table below to find the amount of water that will drip from the faucet in a number of hours:

Number of hours	**10**	**2 ½**	**5**	**8**	**24**			**36 ¾**
Amount of water (in liters)	**200**					**180**	**325**	

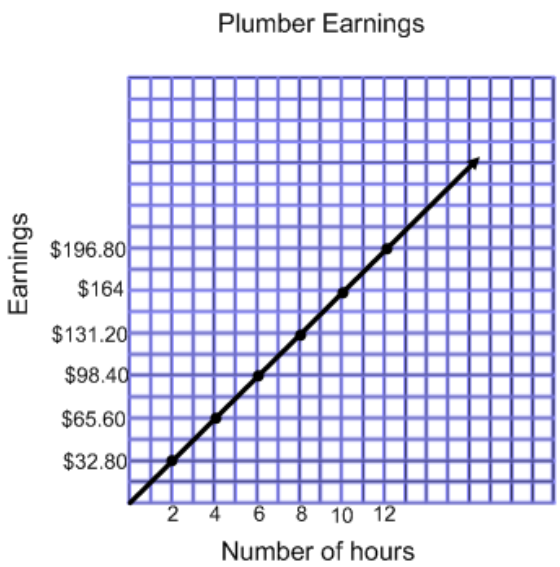
D) Mr. Galileo has a sink in his room that is also clogged. The faucet in his room is dripping at a constant rate. Mr. Galileo observed that in 10 minutes, two and a half liters of water were in the sink. Whose sink is filling faster, Ms. Picasso or Mr. Galileo? Explain.

E) Mr. Galileo had to wait for the plumber to finish working in Ms. Picasso's room. When the plumber arrived, Mr. Galileo found that there were 67 ½ liters of water in the sink. How long did Mr. Galileo have to wait for the plumber? Show your work.

F) Before unclogging the sink, the plumber had to empty the water from the sink. The plumber used a bucket that could hold 3 ¾ liters. If there are 67 ½ liters of water in the sink, how many times did the plumber have to fill the bucket to empty the sink?

G) The graph to the right represents how much the plumber earns while working. How much will he earn after working 21 ¼ hours? Justify your answer.

H) The plumber would like to know his earnings (*e*) for any number of hours (*h*). Write an equation to find his earnings for any number of hours.



Resources

For students:

http://www.internet4classrooms.com/grade_level_help/number_operations_seventh_7th_grade.htm (Link to rate/ratio)

http://www.onlinemathlearning.com/proportion-problems.html (Proportion word problems)

http://math.rice.edu/%7Elanius/proportions/quiz1.html (All about ratios quiz)

http://math.rice.edu/%7Elanius/proportions/ (Links for ratio practice)

http://www.virtualnerd.com/pre-algebra/ratios-proportions/reduce-to-check-proportionality.php (Step-by-step video of how to determine whether two ratios are proportional)

www.virtualnerd.com (Videos demonstrating mathematical skills and concepts)

For teachers:

https://mathematicsteachingcommunity.math.uga.edu/files/105383401-RatioSlides-Kansas-2012-09.pdf

http://insidemathematics.org/index.php/ratios-and-proportional-relationships-rp

http://www.cbmsweb.org/Forum4/Drafts/A.06_Ratios&ProportionalRelationships-MiddleSchool.pdf

www.virtualnerd.com (Videos demonstrating mathematical skills and concepts)

ELL Strategies and Rationale by Modality

Modality	List of Strategies	Rationale
Reading	Linguistic Modification ➢ Minimize use of unfamiliar non-math vocabulary (either through elimination or pre-teaching) ➢ Avoid using idioms unless they are explained or used as a teaching tool (i.e., informal vs. formal t-chart) ➢ Provide signposts that refer to prior examples or problems ➢ Use imperatives in place of question phrases when helpful ("What is the closest estimation of the price?" becomes "Estimate the price to the nearest cent.") ➢ Use active rather than passive voice/verbs (i.e., "Ana is seen out the window…" becomes "You see Ana from the window.") ➢ Break sentences with conditional phrases down into separate sentences (i.e., "If Ana is driving her car 50 miles an hour and has been driving for 2 hours, what distance has she traveled?" becomes "Ana is driving 50 miles an hour. She has been driving for 2 hours. What distance has she traveled?") Use of native language and/or glossaries http://www.p12.nysed.gov/biling/bilinged/bilingual_glossaries.htm Pre-Teaching/Reinforcing Essential Vocabulary ➢ Venn Diagrams ➢ Frayer Models ➢ Word Banks ➢ Concept Maps ➢ Tables Building Background/Activating Prior Knowledge ➢ Create small-group instruction plans for students who may not yet have prerequisite skills. ➢ Provide photographs, illustrations, and other graphic elements ➢ Create outlines of word problems that simplify text while preserving content rigor ➢ Use video clips and think-alouds to build schema Use informal and formal assessments	Word problems can be challenging for ELLs, both with respect to vocabulary and grammar in a language that they are still acquiring. Access for ELLs can be increased through both embedded and explicit language teaching and support. ELLs can benefit from linguistic modification of materials that preserve the content of tasks presented. Keep in mind that modification should amplify not simplify (Walqui 2006) materials. While it is preferred that ELLs engage in language learning through their exploration and mastery of content, at times explicit teaching is needed. These decisions should be made based on individual language proficiency data, including but not limited to NYSESLAT modality scores, input from other teachers, and formative and summative assessments.

Writing	Math journals Have students justify solutions in writing using targeted vocabulary Provide opportunities for practice using sentence frames Model the intended product with exemplars of student work Map language forms to language functions in lesson objectives ➢ Describing Space/Time Relations – prepositional phrases ➢ Making Predictions – verbs (future and conditional tense) ➢ Comparing and Contrasting – adjectives, comparative/superlative, adverbs ➢ Sequencing – subordinate conjunctions (e.g., after, before, whenever, unless), time adverbs Building Background/Activating Prior Knowledge ➢ Allow use of native language for brainstorms and provide students with glossaries if possible. ➢ Create small-group instruction plans for students who may not yet have prerequisite skills. ➢ Clarify learning goals and expectations (e.g., KWL charts, peer- and self-assessments, exit tickets) Promoting Higher-Order Thinking Skills Through Questioning Examples of Questions to Ask Before Solving a Problem ➢ What is the problem asking me to do? ➢ Can I represent this problem with manipulatives? Diagrams? Equations? ➢ Can I solve this problem with manipulatives? Diagrams? Equations? ➢ How do I know if I have enough information? ➢ Is there any information I can eliminate? ➢ What is my first step? Examples of Questions to Ask After Solving a Problem ➢ How can I explain my solution? Can I describe the steps I used? ➢ How can I check my work? ➢ Can I solve this problem a different way? ➢ How is this problem like another problem I solved? How is it different? ➢ What mistakes might someone make in solving this problem? ➢ Do I need more information to solve this problem? How can I prove it?	Language functions are verbs related to expected outcomes. Understanding of and some fluency with these are needed to engage in academic tasks and must often be explicitly taught to ELLs. For example, the function of comparing may also require support and/or explicit teaching in the use of adjectives, comparatives and superlatives. ELL students benefit from linguistic routines. One beneficial scaffold is providing students with open-ended questions to ask before and after solving problems. These can be provided as reference sheets or be used to develop sentence frames.

Speaking/ Listening	Think-Pair-Share Informal vs. Formal Vocabulary T-Charts Modeling through rephrasing student talk using formal vocabulary Elicit student discussion about ideas that come up during lesson through strategies such as turn and talk and think-pair-share Have students verbally justify solutions using targeted vocabulary Sentence Frames Implementation Example ➢ Here is the first sentence frame. Let's read it together. ➢ Does anyone have a question about this sentence? Are there any words that we don't know or understand? (Responses can be charted) ➢ Do we agree or disagree with the sentence? Why? ➢ Let's think about some numbers/expressions/equations/words that would make this sentence true. ➢ Let's think about some numbers/expressions/equations/words that would make this sentence false. ➢ Share your sentence with a partner or table group to check for accuracy. Structured Academic Discussion	Both critical thinking skills and content knowledge in students' primary languages can transfer to the new language given structured opportunities. Comprehensible output scaffolds help students speak accountably and effectively about mathematics. By using academic language, students can demonstrate their proficiency with content and vocabulary. General sentence frames and sentence starters can be displayed around the room, on bookmarks or reference sheets for students. Sentence frames as well as structured discussion outlines can be shared with students to model how they will use them in discussing the problems they are engaged in.