The Scholars' Academy

Mission Statement: To Prepare for College and Careers of the 21st Century
"Technology Today, Smarter Tomorrow: Removing Time and Place from Teaching and Learning"
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Welcome to College Calculus
Summer Performance Based Task

Focus Question: What are the real life applications of polynomials and function behavior?

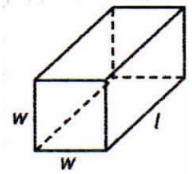
Course: College Calculus

Due Date: September 10th

Key Common Core Standards:

F-IF.4 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
F-IF.7A Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.

Task Overview:
The United States Postal Service will not accept rectangular packages if the perimeter of one end of the package is greater than 130 in. Consider a rectangular package with square ends as shown in the figure.



What are some possible measurements for an acceptable United States Postal Service rectangular package with square ends? What is the maximum volume that fits the size constraints?

Task Steps & Requirements:
Your task is to generate and analyze the possible dimensions of United States Postal Service allowable packages and their resulting functions. Complete the following questions on a **separate sheet of paper**. Some questions may also require graph paper. Use the Internet or other resources to research what you do not remember from your earlier math courses.

Part 1 - Postal Package Measurements
a) Assume that the perimeter of one end of the package plus the length of the package equals the maximum 130 inches. Complete the table with some possible measurements for the length and width of the package. Then find the corresponding volume of each package.

Width (inches)	Length (inches)	Volume (inches ^{3})
10	90	9000

b) Give an estimate for the largest possible volume of an acceptable United States Postal Service rectangular package with square ends.

Part 2 - Postal Package Polynomial Function

a) Using the package described in part 1a, write an expression for l, the length of the package, in terms of w, the width of the square ends of the package.

b) Write the volume of the package V as a function of w, the width of the square ends of the package.

c) Consider the smallest and largest possible values for w that makes sense for the function you wrote in part 2b. Give the domain of the function as a model of the volume of the postal package.

Part 3 - Postal Package Graph

a) Sketch a graph of the function in part 2b over the domain that you found in part 2c. Include the scale on each axis.

b) Use a graphing calculator to find the coordinates of the maximum point of the function that you graphed.

c) What information do the coordinates of the maximum point of the function found in part 3b provide with respect to an acceptable United States Postal Service package with square ends?

Part 4 - Postal Package Function & Graph Analysis

a) Write a polynomial function $f(x)$ defined over the set of real numbers such that it has the same function rule as $V(w)$ the rule you found in part 2b. Sketch the graph of the function.

b) Name any relative maximum values and relative minimum values of the function $f(x)$ in part 4a.

c) Name any x-intercepts or y-intercepts of the function $f(x) = -4x^3 + 130x^2$

d) Examine the end behavior of $f(x) = -4x^3 + 130x^2$

As x goes to ∞ (gets larger towards positive x-values), what behavior does the function have?
As x goes to $-\infty$ (gets larger towards negative x-values), what behavior does the function have?

e) Examine the end behavior of $f(x) = 3x^2 - 6$

As x goes to ∞ (gets larger towards positive x-values), what behavior does the function have?
As x goes to $-\infty$ (gets larger towards negative x-values), what behavior does the function have?

f) In a well-developed paragraph, answer the following:

- What patterns do you notice about the end behavior of polynomial functions?
- How does the degree of the polynomial affect the end behavior?
- How does the sign of the leading coefficient affect the end behavior?