COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 8 MATH: REPRESENTING AND INTERPRETING PROPORTIONAL RELATIONSHIPS

UNIT OVERVIEW

This unit builds directly on work on proportional reasoning and the use of expressions and equations from 6th and 7th grade and extends the ideas more formally into the realm of algebra. This unit continues to deepen students' abilities to model real-world mathematical problems with graphs, tables, and equations; to understand the connections among these models; and to use these models to better understand problems.

TASK DETAILS

Task Name: Representing and Interpreting Proportional Relationships

Grade: 8

Subject: Mathematics

Depth of Knowledge: 3

Task Description: The sequence of tasks asks students to represent and interpret proportional relationships graphically and algebraically.

Standards Assessed:

7.RP.2 Recognize and represent proportional relationships between quantities.
7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b.
8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.
MP.2 Reason abstractly and quantitatively.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.

NYC Department of Education
Dennis M. Walcott, Chancellor

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement Common Core-aligned tasks that are embedded in a unit of instruction. We have learned through our pilot work that focusing instruction on units anchored in rigorous Common Core-aligned assessments drives significant shifts in curriculum and pedagogy. Call out boxes and Universal Design for Learning (UDL) support are included to provide ideas around how to include multiple entry points for diverse learners.

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Input was given from the Academic Evaluation Team in the DOE Central Office and the Common Core Fellows.

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WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 8 MATH: REPRESENTING AND INTERPRETING PROPORTIONAL RELATIONSHIPS

PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

1. Computer Games

Mo'Nique discovered that four computer games cost $92. She built the following table to help her think through a problem, but didn't know what to do next.

Games	Cost
4	92
8	184
12	276
16	368

a. Create a scatterplot to graph the data in the table above.

b. Find the slope of the line that passes through the scatterplot, and explain in words what the slope means in the context of the problem.

c. Explain in words how you can use the slope to solve Mo'Nique's problem: How many computer games will cost $575?

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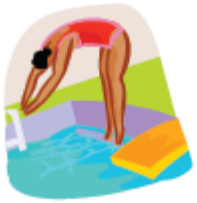
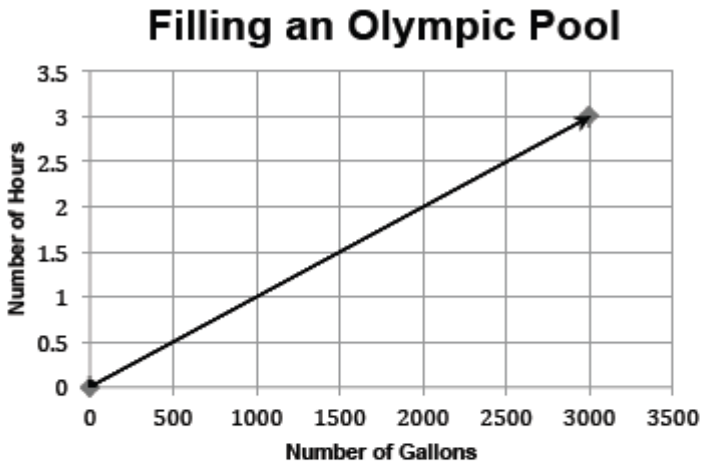
2. Olympic Pool

In the graph below, an Olympic-sized swimming pool is being filled with an average-sized garden hose.





a. At what hourly rate is the pool in the graph above being filled? Use mathematical reasoning to justify your response.

b. Is the hourly rate at which the pool is filling the same as the slope of the line? Why or why not? Explain your reasoning in words.

c. Determine the equation of the line in the graph above. Show how you determined your answer.

d. How long will it take to fill the 660,000-gallon pool? Use mathematical reasoning to justify your response.

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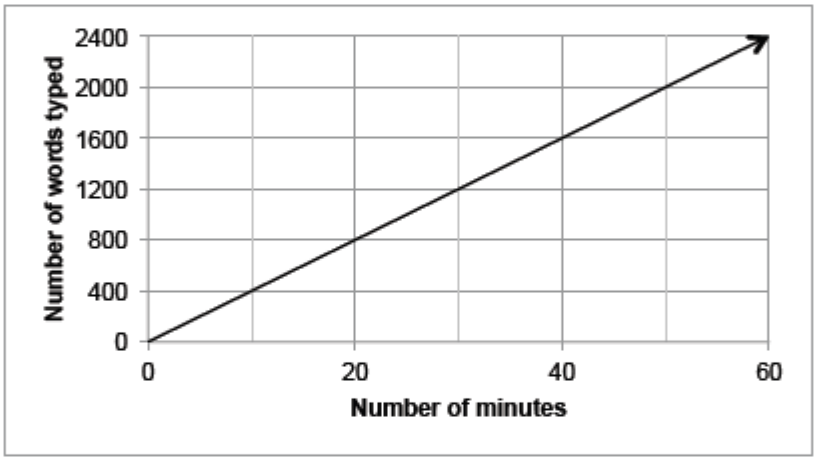
4. Professional Typist

A professional typist can typically type 50 words per minute.

a. Graph the relationship described above.

b. Write an equation relating the number of words typed (w) in terms of the time (t) in minutes. What information do the numbers in the equation represent? Explain your reasoning in words.

c. The graph below shows the number of words typed by Donald over a 60-minute period of time. Write an equation relating the number of words typed in terms of the time in minutes. Use mathematical reasoning to justify your response.



d. Determine if the following statements are true or false. Provide evidence based on the context of the problem that shows why the statements are true or false. Explain your reasoning in words.

i) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's graph to the professional's graph.

ii) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's equation to the professional's equation.

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GRADE 8 MATH: REPRESENTING AND INTERPRETING PROPORTIONAL RELATIONSHIPS

RUBRIC

This section contains benchmark papers that include student work samples for each of the four tasks in the INTERPRETING PROPORTIONAL RELATIONSHIPS GRAPHICALLY AND ALGEBRAICALLY assessment. Each paper has descriptions of the traits and reasoning for the given score point, including references to the Mathematical Practices.

institute for learning

Scoring Guide

1. Computer Games

Mo'Nique discovered that four computer games cost $92. She built the following table to help her think through a problem, but didn't know what to do next.

Games	Cost
4	92
8	184
12	276
16	368

a. Create a scatterplot to graph the data in the table above.

b. Find the slope of the line that passes through the scatterplot, and explain in words what the slope means in the context of the problem.

c. Explain in words how you can use the slope to solve Mo'Nique's problem: How many computer games will cost $575?

Eighth Grade CCSS for Mathematical Content

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure

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Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathemat understanding. The student's strategy and execution meet the content (including concepts, techniq representations, and connections), thinking processes and qualitative demands of all parts of the task. Mi omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may present, but no errors of reasoning appear.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly uses numbers from the table to graph. Abstracts numbers from the graph and/or table; creates a representation for slope, solves the problem and correctly notes the meaning of the results in the context of the problem.	Response indicates the ability to graph proportional relationships and interpret the unit rate as the slope of the graph. Correctly graphs the data, determines the slope indicated in the graph or table and identifies the slope as the unit rate or cost per game. Strategies for determining price include: a. Noting that, since the scatterplot can be modeled by a straight line passing through (0, 0), the relationship is proportional, so dividing $\frac{92}{4}$ will determine both the unit rate (cost per game) and the slope of the line.
Construct viable arguments and critique the reasoning of others: Justifies answer to part c using appropriate mathematical reasoning, clearly explaining the strategy used (graph, table, proportion, equation). Uses multiple data points (see a and b to right), tests for proportionality or claims linearity (see c, d and e to right) to establish $23 as the constant rate of change and the unit price per game.	b. Forming the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ between the data in the table and noting that each is equivalent to $\frac{23}{1}$, indicating that the cost per game is $23. c. Forming the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ between two data points in the table and noting that change is constant between all other data points, indicating that the cost per game is $23. d. Noting that, since the scatterplot can be modeled by a straight line passing through (0, 0), the relationship is proportional, so the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ is constant, and equal to $\frac{23}{1}$, indicating that the cost per game is $23.
Model with mathematics: Writes number sentences describing the reasoning used, e.g., $\frac{92}{4} = \frac{23}{1}$, writes equivalent fractions or a proportion. Possibly writes C = $23g, where C is the dollar cost for g games.	e. Showing $\frac{rise}{run}$ "steps" on the scatterplot; simplifying the resulting ratios to $\frac{23}{1}$, indicating that the cost per game is $23. f. Using unit rate to determine the equation C = $23g, where C is the dollar cost for g games and 23 is the slope of the scatterplot or constant of proportionality or unit rate in dollars per game.

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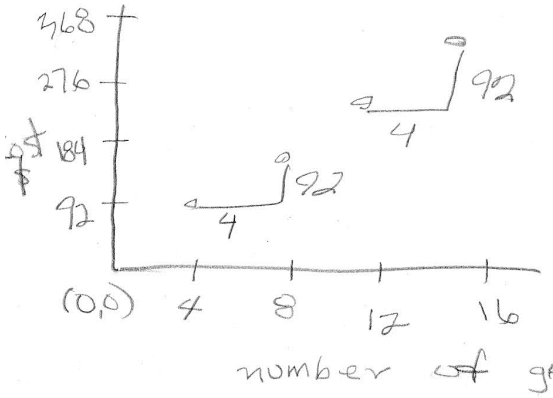
Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, labels quantities, etc. and appropriately labels the final results as cost in dollars and number of games.	Response indicates the ability to use the unit rate/slope to solve problems. Determines the slope (unit rate) indicated in the graph or table; uses that information to determine the number of games that can be bought for $575. Strategies for determining price include: g. Dividing $575 by 23. h. Extending the table or graph to the point where cost = $575, and reading the number of games indicated there. i. Scaling up (1, 23) using fractions, ratios, table or proportions to (25, 575). j. Solving $575 = 23g for g games.
Look for and make use of structure: Uses the multiplicative structure of proportional relationships to scale up OR uses unit rate, OR correctly uses the structure of linear equations to solve the problem.	Response to part c indicates the ability to "use stated assumptions, definitions, and previously established results in constructing arguments…and build a logical progression of statements."



I didn't connect the ~~dots~~ cuz I can't buy PART of A game.

b. slope = $\frac{\text{rise}}{\text{run}} = \frac{92}{4} = \frac{23}{1}$ so every computer game costs $23

c. One game is $23. Mbigoe should divide 23 into $575 and then she'll know that 25 games cost $575.

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Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathemat understanding. The student's strategy and execution meet the content (including concepts, techniq representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of necessary connections between the context and the ratio may be missing.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution*, but indicates a struggle to interpret parts of the context correctly.*

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts numbers from the graph and/or table; creates a representation for slope, solves the problem and correctly notes the meaning of the results in the context of the problem. *Possibly graphs cost on x-axis, and struggles to interpret slope as a result.*	Response indicates the ability to graph proportional relationships. *Possibly graphs cost on x-axis, and struggles to interpret slope as a result (or incorrectly determines slope as $\frac{23}{1}$), or fails to interpret slope as the price per game, BUT NOT BOTH.* Uses any of the 3-point strategies described below. Correctly graphs the data, determines the slope indicated in the graph or table *but fails to interpret slope as the price per game.* Strategies for determining price include: k. Noting that, since the scatterplot can be modeled by a straight line passing through (0, 0), the relationship is proportional, so dividing $\frac{93}{4}$ to find $23. l. Forming the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ between the data in the table and noting that each is equivalent to $\frac{23}{1}$. m. Forming the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ between two data points in the table and noting that change is constant between all other data points. n. Noting that, since the scatterplot can be modeled by a straight line passing through (0, 0), the relationship is proportional, so the ratio of $\frac{\text{change in cost}}{\text{change in number of games}}$ is constant, and equal to $\frac{23}{1}$. a. Shows $\frac{rise}{run}$ "steps" on the scatterplot; simplifying the resulting ratio to $\frac{23}{1}$. b. Using unit rate to determine the equation C = $23g, where C is the dollar cost for g games.
Construct viable arguments and critique the reasoning of others: Justifies answer to part c using appropriate mathematical reasoning, clearly explaining the strategy used (graph, table, proportion, equation). Uses multiple data points (see a and b to right), tests for proportionality or claims linearity (see c, d and e to right) *but fails to establish $23 as the constant rate of change and the unit price per game.*	
Model with mathematics: Writes number sentences describing the reasoning used, e.g., $\frac{92}{4} = \frac{23}{1}$, writes equivalent fractions or a proportion. Possibly writes C = $23g, where C is the dollar cost for g games.	

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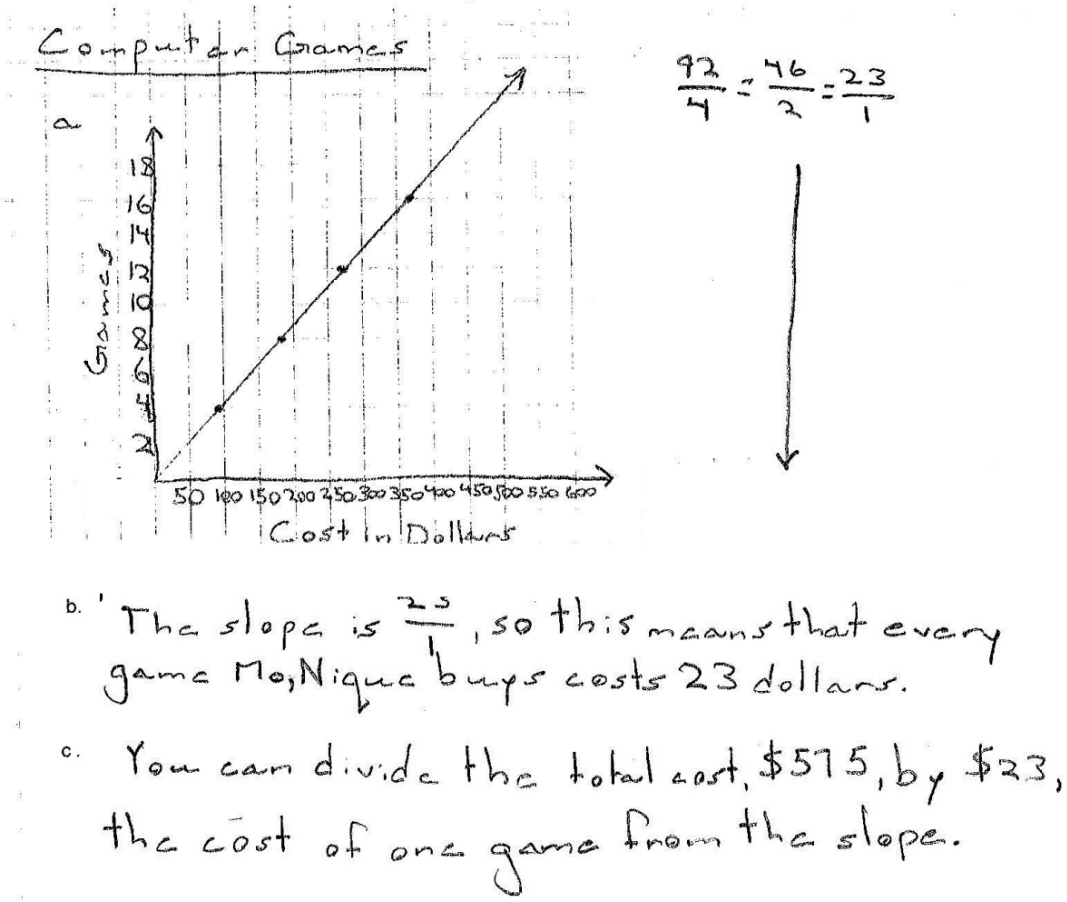
Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, labels quantities, etc. and appropriately labels the final results as cost in dollars and number of games.	Response indicates the ability to use the unit rate/slope to solve problems, *but fails to interpret slope as the price per game.* Determines the slope indicated in the graph or table; uses that information to determine the number of games that can be bought for $575. Strategies for determining price include: a. Dividing $575 by 23. b. Extending the table or graph to the point where cost = $575, and reading the number of games indicated there. c. Scaling up (4, 92) using fractions, ratios, table or proportions to (25, 575). d. Solving $575 = 23g for g games.
Look for and make use of structure: Uses the multiplicative structure of proportional relationships to scale up OR uses unit rate, OR correctly uses the structure of linear equations to solve the problem.	Response to part c indicates the ability to "use stated assumptions, definitions, and previously established results in constructing arguments…and build a logical progression of statements," *but omits the logical progression step of noting that $23 is the cost per game.*



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One-Point Score

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the ratio are missing.

Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the context.

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts numbers from the graph and/or table; creates a representation for slope, solves the problem and correctly notes the meaning of the results in the context of the problem. *Possibly graphs cost on x-axis.*	Response indicates the ability to graph proportional relationships *but fails to interpret the unit rate as the slope of the graph and does not indicate the ability to use the unit rate/slope to solve problems.* Attempts to reason in any of the following ways: a. *Noting the rate of change from the table or graph, but failing to understand how to use that information to answer parts b and c.* b. *Appropriately scaling up/down the data from (4, 92), but failing to understand how to use that information to answer parts b and c.* c. Correctly determining unit rate, *possibly by dividing $92 by 4, but failing to recognize 23 as the slope and/or the price per game.* d. Determining the equation C = $23g, where C is the dollar cost for g games, *but failing to understand how to use that information to answer parts b and c.* Response to part c may indicate extending the table or graph, *but fails to use slope or unit rate to answer the problem.* *Response to part c does not indicate the ability to "use stated assumptions, definitions, and previously established results in constructing arguments…and build a logical progression of statements."*
Construct viable arguments and critique the reasoning of others: *Fails to argue logically, using determined information. May argue on illogical grounds, e.g., 575/92 gives a fractional number of games.*	
Model with mathematics: Writes number sentences describing the reasoning used, e.g., $\frac{92}{4} = \frac{23}{1}$, writes equivalent fractions or a proportion. Possibly writes C = $23g, where C is the dollar cost for g games, *but fails recognize how to use that information to answer parts b and/or c.*	
Attend to precision: Possibly multiplies, scales up, creates equivalent fractions, label quantities, etc., *but fails to appropriately label the final results as cost in dollars and number of games.*	
Look for and make use of structure: *Fails to understand or make use of the structure of slope or unit rate to solve the problem.*	

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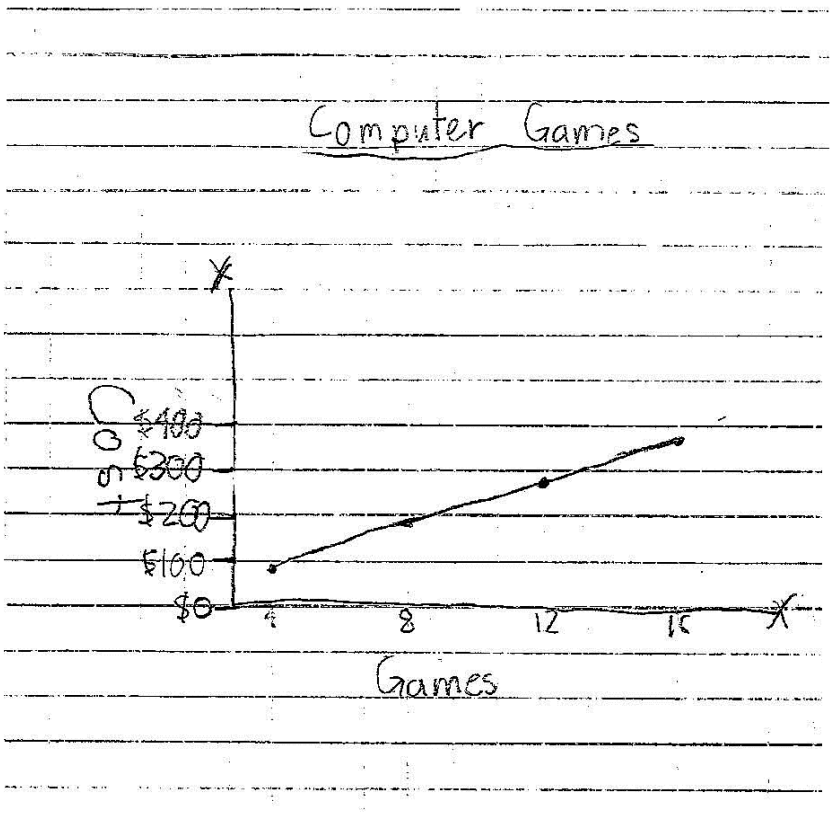

b. The slope is rise over run and is also a straight line so it will keep going depending on however many games.

c. You can continue to draw the line until it reaches $575 and check how many games there are on the X axis.

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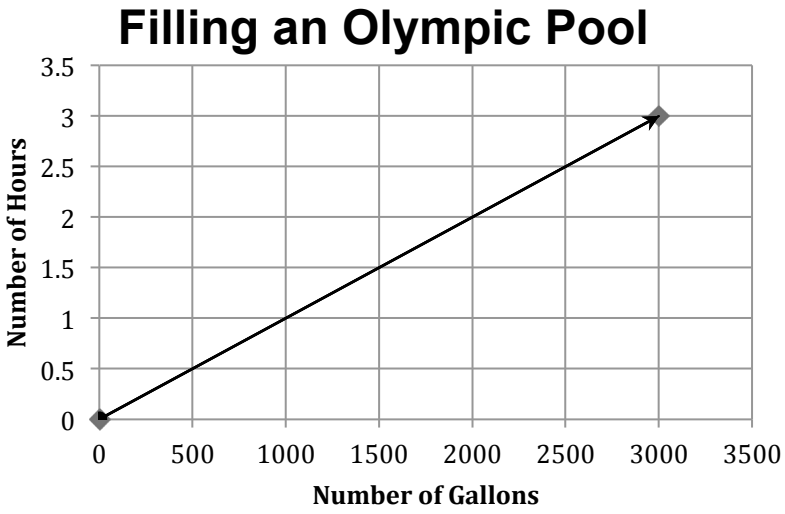
Scoring Guide

2. Olympic Pool

In the graph below, an Olympic-sized swimming pool is being filled with an average-sized garden hose.



a. At what hourly rate is the pool in the graph above being filled? Use mathematical reasoning to justify your response.

b. Is the hourly rate at which the pool is filling the same as the slope of the line? Why or why not? Explain your reasoning in words.

c. Determine the equation of the line in the graph above. Show how you determined your answer.

d. How long it will take to fill the 660,000-gallon pool? Use mathematical reasoning to justify your response.

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Eighth Grade CCSS for Mathematical Content

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

8.EE.6 Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

7.RP.2a Recognize and represent proportional relationships between quantities.

a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

7.RP.2b Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

The CCSS for Mathematical Practice

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

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Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear. The necessary connections between the context and the slope vs. hourly rate are handled appropriately.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Abstracts numbers from the graph; creates a representation for slope and hourly rate at which the pool is being filled, solves the problem and correctly notes the meaning of the results in the context of the problem.	Response indicates the ability to interpret the unit rate in hours per gallon as the slope of the graph, to distinguish the slope of the graph from the hourly unit rate that exists between the same conditions, and to use proportional reasoning to solve problems. Correctly determines the hourly rate at which the pool is being filled, the slope of the graphed line, and distinguishes between the two, i.e., distinguishes between $\frac{1000}{1}$ gallons per hour and $\frac{.001}{1}$ hours per gallon.
Construct viable arguments and critique the reasoning of others: Justifies answer to parts b and d using appropriate mathematical reasoning, clearly explaining the strategy used (graph, table, proportion, equation). Notes and uses the proportional relationship and/or linearity (see a, c, d and e to right) to justify the hourly rate and to solve part d.	Strategies for determining slope of the line include: a. Observing, appropriately testing and stating that the graph represents a proportional relationship; choosing a readable point on the line and dividing, e.g., $\frac{1}{1000}$, to determine the slope. b. Building a table from the readable points on the graph, forming the ratio of $\frac{\text{change in hours}}{\text{change in number of gllons}}$ between the data in the table and noting that each is equivalent to $\frac{1}{1000}$. c. Forming the ratio of $\frac{\text{change in hours}}{\text{change in number of gallons}}$ between two data points on the graph and noting that, since the graph is a straight line, the slope is constant between all other data points, $\frac{1}{1000}$.
Model with mathematics: Writes number sentences describing the reasoning used, e.g., since (1000gal.)(660) = 660,000 gallons, then (1hr.)(660) = 660 hours; writes equivalent fractions or a proportion. Writes H = .001g, where H is the number of hours, and g is the number of gallons of water. Possibly writes G = 1000h, where G is the number of gallons of water and h is the number of hours.	d. Observing, appropriately testing and stating that, since the graph represents a proportional relationship, the ratio of $\frac{\text{change in hours}}{\text{change in number of gallons}}$ is constant, and equal to $\frac{1}{1000}$. e. Showing a $\frac{rise}{run}$ "step" on the graph of the line; simplifying the resulting ratio to $\frac{1}{1000}$. f. Using unit rate to determine the equation H = .001g, where H is the number of hours, g is the number of gallons of water and $\frac{1}{1000}$ represents the slope of the line.

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	Strategies for determining hourly rate at which the pool is being filled include ones similar to a – e above, but with the recognition that the hourly unit rate for filling the pool is $\frac{\text{change in number of gallons}}{\text{change in hours}}$, and is, therefore, equal to 1000 gallons/hour.
Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, labels quantities, etc. and appropriately labels the final results as hours and the variables as number of hours, and number of gallons of water. **Look for and make use of structure:** Uses the multiplicative structure of proportional relationships to scale up OR uses unit rate, OR correctly uses the structure of linear equations to solve the problem. **Look for and express regularity in repeated reasoning:** Uses information gained in parts a and b to shortcut decision-making in parts c and d. Possibly maintains oversight of the process by solving part d in more than one way, or by checking to see if the 660-hour response makes sense in terms of the numbers in the context.	Response to part c indicates the ability to derive the equation y = mx for a line through the origin, and to use the equation or proportional reasoning to solve problems. Strategies for determining the equation of the line include: g. Determining the slope of the line or using the slope determined in part b and the fact that the relationship is proportional (line passes through (0, 0)). h. Using the slope determined in part b and the y-intercept indicated on the graph. Strategies for solving part d include: i. Substituting g = 660,000 into the equation H = .001g, where H is the number of hours, g is the number of gallons of water and evaluating the expression to solve for H. j. Substituting G = 660,000 into the equation G = 1000h, where G is the number of gallons of water and h is the number of hours, and solving the equation. k. Scaling up a readable point on the line (or in the table), using fractions, ratios, multiplication or proportions, e.g., 1000: 1 as 660, 000: 660. l. Extending the graph or table to the appropriate point.

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a. It is being filled at a rate of 1000 gallons per hour.

b. No, because the slope is the change in the y-axis compared to the change in the x-axis. The hourly rate is the change in the x-axis compared to the change in the y-axis.

c. x = 1000 y, because of the 1,000 gallons per hour.

d. It will take 660 hours because if you divide 660,000 by the 1,000 gallons it fills up in one hour, it will take 660 hours to fill up the 660,000 gallon pool.

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Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the slope vs. hourly rate may be missing.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution*, but indicates a struggle to interpret parts of the context correctly.*

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Abstracts numbers from the graph; creates a representation for slope and hourly rate at which the pool is being filled, solves the problem and correctly notes the meaning of the results in the context of the problem. *Struggles to distinguish slope of the graph from hourly unit rate.*	Response indicates the ability to interpret a unit rate as the slope of the graph, and to use proportional reasoning to solve problems, *but struggles to distinguish slope from hourly unit rate* given the same conditions. Correctly determines the slope of the graphed line, OR the hourly rate at which the pool is being filled, *but fails to distinguish between the two.* Uses any of the 3-point strategies described below.
Construct viable arguments and critique the reasoning of others: Justifies answer to parts b and d using appropriate mathematical reasoning, clearly explaining the strategy used (graph, table, proportion, equation). Notes and uses the proportional relationship and/or linearity (see a, c, d and e to right) to justify the slope OR hourly rate and to solve part d, *but fails to distinguish between the slope and the hourly rate.*	a. Observing, appropriately testing and stating that the graph represents a proportional relationship; choosing a readable point on the line and dividing, e.g., $\frac{1}{1000}$, to determine the unit rate. b. Building a table from the readable points on the graph, forming the ratio of $\frac{\text{change in hours}}{\text{change in number of gallons}}$ between the data in the table and noting that each is equivalent to $\frac{1}{1000}$. c. Forming the ratio of $\frac{\text{change in hours}}{\text{change in number of gallons}}$ between two data points on the graph and noting that, since the graph is a straight line, the slope is constant between all other data points, $\frac{1}{1000}$.
Model with mathematics: Writes number sentences describing the reasoning used, e.g., since (1000gal.)(660) = 660,000 gallons, then (1hr.)(660) = 660 hours; writes equivalent fractions or a proportion. Writes H = .001g, where H is the number of hours, and g is the number of gallons of water. *Possibly writes G = 1000h, where G is the number of gallons of water and h is the number of hours*.	d. Observing, appropriately testing and stating that, since the graph represents a proportional relationship, the ratio of $\frac{\text{change in hours}}{\text{change in number of gallons}}$ is constant, and equal to $\frac{1}{1000}$. e. Showing a $\frac{rise}{run}$ "step" on the graph of the line; simplifying the resulting ratio to $\frac{1}{1000}$. f. Using unit rate to determine the equation H = .001g, where H is the number of hours, g is the number of gallons of water and $\frac{1}{1000}$ represents the slope of the line.

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	Strategies for determining hourly rate at which the pool is being filled include ones similar to a – e above, but uses $\frac{1000}{1}$ or $\frac{\text{change in number of gallons}}{\text{change in hours}}$. *Fails to distinguish this unit rate from the slope of the line.*
Attend to precision: Correctly multiplies, scales up, creates equivalent fractions, labels quantities, etc., and appropriately labels the final results as hours and the variables as number of hours, and number of gallons of water.	Response to part c indicates the ability to derive the equation y = mx for a line through the origin, and to use the equation or proportional reasoning to solve problems. *Possibly writes the equation G = 1000h, where G is the number of gallons of water and h is the number of hours*, *based on response to parts a and b.*
Look for and make use of structure: Uses the multiplicative structure of proportional relationships to scale up OR uses unit rate, OR correctly uses the structure of linear equations to solve the problem.	Strategies for determining the equation of the line include: g. Determining the slope of the line or using the slope determined in part b and the fact that the relationship is proportional (line passes through (0, 0)). h. Using the slope determined in part b and the y-intercept indicated on the graph.
Look for and express regularity in repeated reasoning: Uses information gained in parts a and b to shortcut decision-making in parts c and d. Possibly maintains oversight of the process by solving part d in more than one way, or by checking to see if the 660-hour response makes sense in terms of the numbers in the context.	Strategies for determining solving part d include: i. Substituting g = 660,000 into the equation H = .001g, where H is the number of hours, g is the number of gallons of water and evaluating the expression to solve for H. j. Substituting G = 660,000 into the equation G = 1000h, where G is the number of gallons of water and h is the number of hours, and solving the equation. k. Scaling up a readable point on the line (or in the table), using fractions, ratios, multiplication or proportions, e.g., 1000: 1 as 660, 000: 660. l. Extending the graph or table to the appropriate point.

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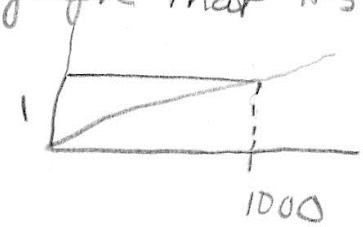
a. the hourly rate is how many gallons each hour. On the graph that is where 1 hour shows 1000 gal, so it's 1000 gal/ 1 hour.

b. You can see on the graph that it's 1 hour for 1000 gal.



c. G = 1000h because it's 1000 gal/hr

d. 1 hr fills 1000 gal

100 hr fills 100,000 gal

600 hr fills 600,000 gal

60 hr fills 60,000 gal

so 660 hr fills 660,000 gal

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One-Point Score

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the slope vs. hourly rate are missing.

Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the context.

Mathematical Practices	Evidenced-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts numbers from the graph; may create a representation for slope or unit rate; may solve part d using proportional reasoning or by extending a table or the graph. *May have difficulty abstracting information from the graph because of scale. i.e., may read the graph incorrectly.*	Response indicates the ability to interpret a unit rate as the slope of the graph, OR to use proportional reasoning to solve problems. Attempts to reason in any of the following ways: a. *Determining the slope of the line and/or the hourly rate, but failing to understand how to use that information to answer parts c and d.* b. *Appropriately scaling up/down the data from* 1:1000 to 660: 660,000 or 1000:1 to 660, 000: 660, *but failing to determine unit rates or the equation of the line*. c. Determining the equation H = .001g, where H is the number of hours, and g is the number of gallons of water or G = 1000h, where G is the number of gallons of water and h is the number of hours, *but failing to distinguish the slope of the graph from the hourly rate and failing to understand how to use the equation to answer part d..* Response to part d may indicate extending a table or graph, *but fails to use slope, unit rate or an equation to answer the problem.*
Construct viable arguments and critique the reasoning of others: Justifies correct responses using appropriate mathematical reasoning, clearly explaining the strategy used (graph, table, proportion, equation). *Fails, however, to successfully respond to all parts of the problem.*	
Model with mathematics: Writes number sentences describing the reasoning used, e.g., since (1000gal.)(660) = 660,000 gallons, then (1hr.)(660) = 660 hours; writes equivalent fractions or a proportion. *May write the equation H = .001g, where H is the number of hours, and g is the number of gallons of water or G = 1000h, where G is the number of gallons of water and h is the number of hours.*	
Attend to precision: Possibly multiplies, scales up, creates equivalent fractions, label quantities, etc., *but fails to appropriately label the final results as hours or the variables as number of hours, and number of gallons of water.*	
Look for and make use of structure: *Fails to understand or make use of the structure of slope to solve the problem.*	
Look for and express regularity in repeated reasoning: *Fails to use information gained in parts a and b to shortcut decision-making in parts c and d. Possibly fails to maintain oversight of the process by solving part d in more than one way, or by checking to see if part d's response makes sense in terms of the numbers in the context*	

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a. For every half hr 500 gal.'s of water are being put in the pool.

b. Yes they are the same because they are both 500 gal.'s per half hr. Which means they are the same.

c.

d. It will take 1,220 hrs to fill a 660,000 gal. pool. I got my answer by multiplying.

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Scoring Guide

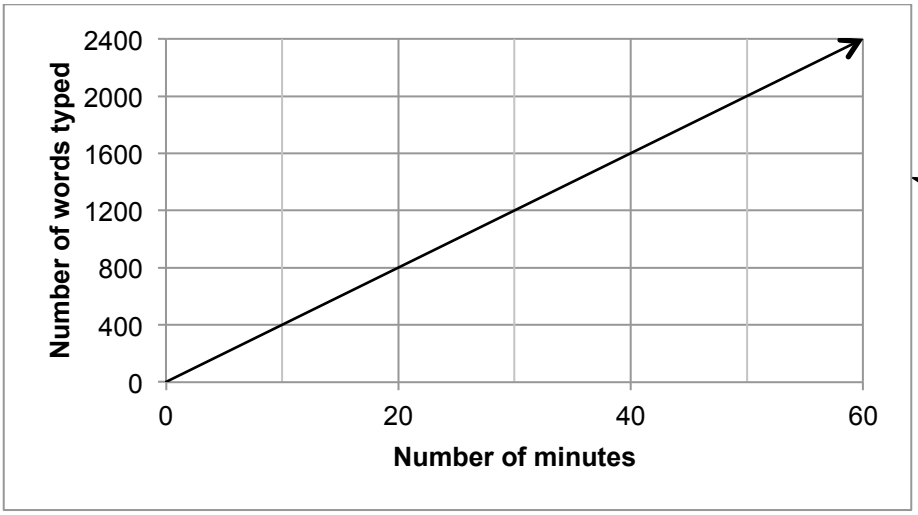
4. Professional Typist

A professional typist can typically type 50 words per minute.

a. Graph the relationship described above.

b. Write an equation relating the number of words typed (w) in terms of the time (t) in minutes. What information do the numbers in the equation represent? Explain your reasoning in words.

c. The graph below shows the number of words typed by Donna over a 60-minute period of time. Write an equation relating the number of words typed in terms of the time in minutes. Use mathematical reasoning to justify your response.



d. Determine if the following statements are true or false. Provide evidence based on the context of the problem that shows why the statements are true or false. Explain your reasoning in words.

i) A person can decide if Donna types faster or slower than the professional typist by comparing Donna's graph to the professional's graph.

ii) A person can decide if Donna types faster or slower than the professional typist by comparing Donna's equation to the professional's equation.

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Eighth Grade CCSS Standards for Mathematical Content

Understand the connection between proportional relationships, lines, and linear equations.

8.EE.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.

Use functions to model relationships between quantities.

8.F.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

The CCSS for Mathematical Practices Addressed in This Task

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning.

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Three-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Minor omissions may exist, but do not detract from the correctness of the response. Minor arithmetic errors may be present, but no errors of reasoning appear. Unit rate or rate of change is appropriately handled in multiple representations.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts 50 words per minute from the context, and 40 words per minute from the graph, recognizing them as unit rates of change; creates the required representations, solves the problem and correctly notes the meaning of the results in the context of the problem.	Response indicates the ability to graph proportional relationships, to construct an equation to model a proportional relationship from both words and a graph describing the relationship, and to interpret the slope of the graph as a unit rate (words per minute) which permits comparison between the relationships. Correctly graphs the data, determines the slope indicated in the graph or description of the relationship and identifies the slope as the unit rate or number of words per minute. Strategies for graphing include: a. Setting up a table of values and graphing the values. b. Recognizing that "50 words per minute" indicates the ratio of $\frac{50 \text{ words}}{1 \text{ minute}}$ is the slope of the graph, then using a $\frac{\text{rise}}{\text{run}}$ "step" process to graph c. Noting that $\frac{50 \text{ words}}{1 \text{ minute}} = \frac{500 \text{ words}}{10 \text{ minute}}$ (or some other equivalent value) and connecting those two points. d. Noting that the relationship is proportional and passes through (0, 0); connecting that point and (1, 50) or (10, 500), etc. Correctly interprets 50 words per minute to be the unit rate and thus slope of a proportional relationship. Strategies for determining the equation of the line include: e. Noting that the relationship is proportional and passes through (0, 0); acknowledging that 50 words per minute is the unit rate and thus the slope of a proportional relationship, y = kx. f. Determining the slope and intercept from either the table or graph and forming the equation. Response to part c may include strategies as described above in f; possibly creates a table of values from the graph.
Construct viable arguments and critique the reasoning of others: Justifies answer to part d using appropriate mathematical reasoning, explaining that the number of words per minute is readily visible in each equation (the coefficient of the variable); or, in the case of the graph, when graphed on the same grid, while the actual number is not readily visible, the visual of a "higher" slope is associated with a larger number of words per minute. May argue that graphs can be misleading if the graphs are separate, and the scales do not coincide.	
Model with mathematics: Uses graphs and equations to model the relationships between the number of words and the elapsed time in minutes.	

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Attend to precision: Chooses appropriate scales for the graph, and graphs correctly; labels graphs and quantities appropriately.	Response to part d indicates the ability to recognize the slope of the graph of a proportional relationship as the unit rate (or constant rate of change), to recognize the coefficient of the variable as the unit rate in the equation y = kx, and to be able to use that unit rate as a basis of comparison in both graphs and equations. May or may not directly state that Donna is slower than a typical typist; agrees that both statements are true by: a. Arguing that, when graphed on the same grid or on grids where the scales coincide, the slope of Donna's graph is "lower" than that of a typical typist, and that a lower slope indicates a smaller unit rate or number of words per minute; or that the slope of the graph of a typical typist is "higher" than Donna's, and that a higher slope indicates a larger unit rate or number or words per minute. b. Determining the slopes (or using them from the response to parts a – c) to argue that 50 words per minute (professional typist) is faster than 40 words per minute (Donna). c. Noting that, in the equation y = kx, k = the slope of the graph, then presenting an argument similar to a and b above. d. Stating that, since the k in y = kx represents the slope of the graph of the equation, the argument presented for the graphs holds equally true for the equations. e. Noting that, since the k in y = kx represents the constant rate of change, and 50 is greater than 40, the typical typist is faster or Donna is slower.
Look for and make use of structure: Recognizes proportional relationships verbally, in graphs and equations and moves readily among the representations for them.	
Look for and express regularity in repeated reasoning: Notes the calculations used in parts a – c can be used in the argument for part d; possibly notes that the fact that k is the slope of the graph connects parts *ii* to part *i,* thus eliminating the need for a separate argument (shortcut).	

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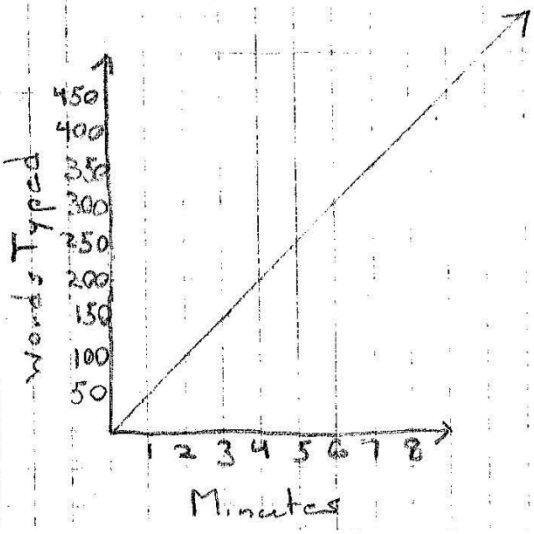
Proffessional Typist

a.

Words	Minutes
50	1
100	2
150	3



b. $50t = w$

The 50 represents the number of words typed in one minute.

c. 800 w in 20 m

$\frac{800 \text{ words}}{20 \text{ min}}$ = 40 words in one minute

$40t = w$

d. Both are True. The rate each minute is the slope of the graph and the coefficient of t. 50 is a steeper slope on a graph than 40 and a larger coefficient.

More words in one minute are typed with 50 than 40.

Two-Point Score

The response accomplishes the prompted purposes and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of all parts of the task. Some of the necessary connections between the context and the unit rate may be missing.

Make sense of problems and persevere in solving them.
Work includes representations of the problem indicating the student made sense of the problem and advanced towards a correct solution*, but indicates a struggle to interpret parts of the context correctly.*

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Correctly abstracts 50 words per minute from the context, and 40 words per minute from the graph, recognizing them as slopes; creates the required representations, and solves the problem. *Struggles to interpret the slope of the graph or the coefficient of the variable as a unit rate (words per minute).*	Response indicates the ability to graph proportional relationships, to construct an equation to model a proportional relationship from both words and a graph describing the relationship; but *struggles to contextualize the slope of the graph or the coefficient of the variable as the unit rate (words per minute).* Uses any of the 3-point strategies described below for graphing*, but may fail to recognize the* $\frac{\text{words}}{\text{minute}}$ *relationship:* a. Setting up a table of values and graphing the values. b. Recognizing that 50 indicates the ratio of $\frac{50}{1}$ is the slope of the graph, then using a $\frac{\text{rise}}{\text{run}}$ "step" process to graph. c. Noting that $\frac{50}{1} = \frac{500}{10}$ (or some other equivalent value) and connecting those two points. d. Noting that the relationship is proportional and passes through (0, 0); connecting that point and (1, 50) or (10, 500), etc. Uses any of the 3-point strategies described below for determining the equation of the line *but may fail to recognize the* $\frac{\text{words}}{\text{minute}}$ *relationship:* e. Noting that the relationship is proportional and passes through (0, 0); acknowledging that 50 is the slope of a proportional relationship, y = kx. f. Determining the slope and intercept from either the table or graph and forming the equation. Response to part c may include strategies as described above in e and f; possibly creates a table of values from the graph.
Construct viable arguments and critique the reasoning of others: Justifies answer to part b, c and/or d using appropriate mathematical *numerical* reasoning, *but struggles to contextualize the slope of the graph or the coefficient of the variable as the unit rate (words per minute), instead using a decontextualized response, i.e., does not connect the response to unit rate (words per minute).*	
Model with mathematics: Uses graphs and equations to model the relationships between the number of words typed and the elapsed time in minutes.	

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Attend to precision: Chooses appropriate scales for the graph, and graphs correctly; labels graphs and quantities appropriately.	Response indicates a *struggle to interpret the slope of the graph or the coefficient of the variable as a unit rate (words per minute), resulting in an inability to compare the rates of the two typists in one of the two representations. May agree with both statements using a decontextualized response, i.e., does not connect the response to unit rate (words per minute).* May or may not directly state that Donna is slower than a typical typist; agrees that *at least one* statement is true, but fails to recognize both are true.
Look for and make use of structure: Recognizes proportional relationships verbally, in graphs and equations and moves readily among the representations for them.	*a.* Arguing that, when graphed on the same grid or on grids where the scales coincide, the slope of Donna's graph is "lower" than that of a typical typist, and that a lower slope indicates a smaller unit rate or number of words per minute; or that the slope of the graph of a typical typist is "higher" than Donna's, and that a higher slope indicates a larger unit rate or number or words per minute, *but failing to agree that part ii is true.* b. Determining the slopes (or using them from the response to parts a – c) to argue that 50 words per minute (professional typist) is faster than 40 words per minute (Donna) *but failing to agree that part ii is true.* *OR* c. Noting that, in the equation y = kx, k = the unit rate, and 50 words per minute (professional typist) is faster than 40 words per minute (Donna), *but failing to agree that part i is true.*
Look for and express regularity in repeated reasoning: Notes the calculations used in parts a – c can be used in the argument for part d; possibly notes that the fact that k is the slope of the graph connects parts *ii* to part *i,* thus eliminating the need for a separate argument (shortcut).	d. Noting that, since the k in y = kx represents the constant rate of change, and 50 words per minute is greater than 40 words per minute, the typical typist is faster or Donna is slower, *but failing to agree that part i is true.* OR e. Arguing that, when graphed on the same grid or on grids where the scales coincide, the "lower" slope is the slower typist and, since in the equation y = kx, k is the slope, the smaller number represents the "lower" slope.

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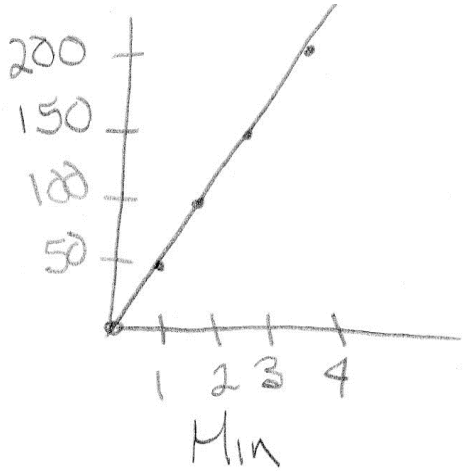
a.

M	W
0	0
1	50
2	100
3	150
4	200



b. W = 50M
Each time it goes up another 50. You can see it on the graph.

c) for Donna $W = \frac{800}{20}M$ $\frac{800}{20}$ is the rise over the run.

d. i. It's True. I put the graph for Donna over mine and hers is lower than mine.

ii. True. Donna's slope is 40. The other is 50, so Donna's doesn't multiply as fast.

One-Point Score

The response accomplishes some of the prompted purposes and communicates the student's mathematical understanding. The student's strategy and execution meet only a few of the content demands of the task. The necessary connections between the context and the ratio are missing.

Make sense of problems and persevere in solving them.
Work is missing representations of the problem or understanding of the constraints of the problem given in the context.

Mathematical Practices	Evidence-Based Traits: Mathematical Content
Reason abstractly and quantitatively: Abstracts 50 words per minute from the context, and may recognize it as a slope; *may incorrectly abstract 10 or as the slope of the graph*; creates the required representations. *Possibly graphs number of words on the x-axis. Struggles to interpret the slope of the graph or the coefficient of the variable as a unit rate (words per minute.). May write decontextualized responses, i.e., does not connect the response to unit rate (words per minute).*	Response indicates the ability to graph proportional relationships. *Possibly graphs number of words on the x-axis, and struggles to compare typists as a result.* Uses any of the 3-point strategies described previously for graphing*, but may fail to recognize the* $\frac{\text{words}}{\text{minute}}$ *relationship.* Response *indicates a struggle* to construct an equation to model a proportional relationship from words and/*or* from a graph describing the relationship. Attempts to reason in any of the following ways: *a. Indicating that the equation of the line of the typical typist is w = t + 50 and/or indicating that the equation of Donna's line is w = t + 40.* *b.* Indicating that the equation of the line of the typical typist is w = 50t, possibly from the words; *suggesting Donna's equation is w = 10t or w = t, from a misreading of the slope of the graph.* c. Indicating that the equation of Donna's line is w = 40t, possibly from the graph; suggesting the equation of the typical typist is w = $\frac{1}{50}$t, *possibly because of graphing the number of words on the x-axis.*
Construct viable arguments and critique the reasoning of others: *Fails to argue logically, using determined information, possibly because of struggle to interpret the slope of the graph and/or the coefficient of the variable as a unit rate (words per minute)*	
Model with mathematics: Uses graphs and equations to model the relationships between the number of words typed in terms of the time in minutes	
Attend to precision: *May or may not choose* appropriate scales for the graph, or graphs correctly; labels graphs and quantities appropriately.	Response indicates a *struggle to interpret the slope of the graph or the coefficient of the variable as a unit rate (words per minute), resulting in an inability to compare the rates of the two typists in one or both of the two representations. May agree/disagree with both statements using a decontextualized response, i.e., does not connect the response to unit rate (words per minute).* Attempts to reason in any of the following ways: *d. Arguing that, since 50 is greater than 40 in the equations w = t + 50 and w = t + 40, you can tell the typical typist is faster; agreeing that you can tell from the graph because the graph of w = t + 50 is "higher" than that of w = t + 40* *e. Arguing that, since 50 is greater than 10 or 1 in the equations w = 50t and w = 10 or w = t, you can tell the typical typist is faster, using either the graphs or the equations or both as the representation.*
Look for and make use of structure: *May or may not recognize proportional relationships verbally, in graphs and equations; fails to move readily among the representations for them.*	
Look for and express regularity in repeated reasoning: *May or may not note the calculations used in parts a – c can be used in the argument for part d.*	

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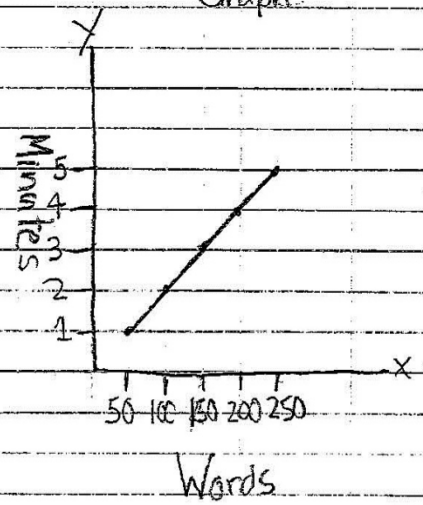
Professional Typist
Table

Words	Minutes
50	1
100	2
150	3
200	4
250	5

Professional Typist
Graph



b. 50 x # of minutes = Total Words

c. 10 x # of minutes = Total Words

d. The faster the typist the steeper the slope.

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GRADE 8 MATH: REPRESENTING AND INTERPRETING PROPORTIONAL RELATIONSHIPS

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points. The student work shows examples of student understandings and misunderstandings of the task.

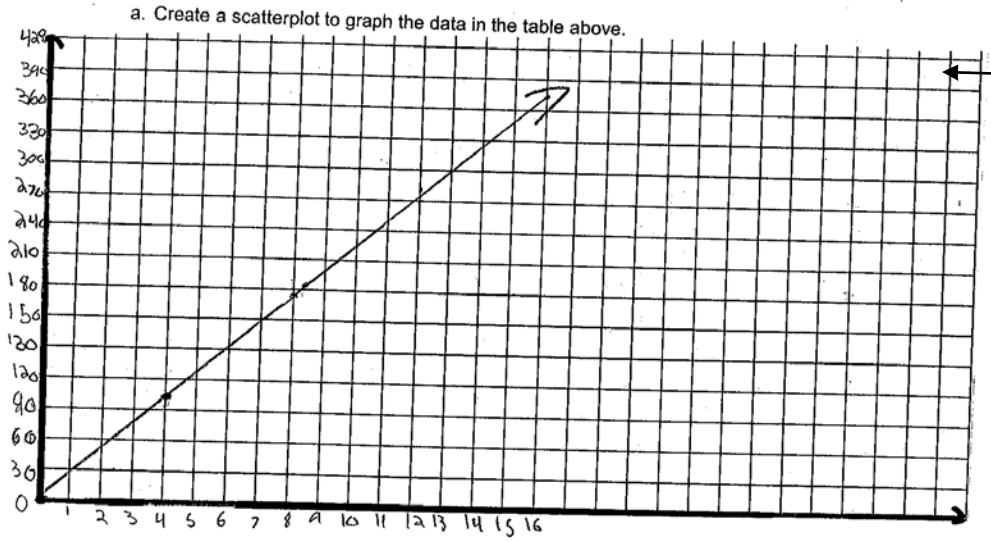
NYC Department of Education
Dennis M. Walcott, Chancellor

Student Work: Score Point 3

Mo'Nique discovered that four computer games cost $92. She built the following table to help her think through a problem, but didn't know what to do next.

Games	Cost
4	92
8	184
12	276
16	368

a. Create a scatterplot to graph the data in the table above.



The student plots the points in the table and connects them to form a line; however, the points are not plotted correctly according to the scale and the axes are not labeled.

b. Find the slope of the line that passes through the scatterplot, and explain in words what the slope means in the context of the problem.

$m = \frac{92}{4} = 23$

The slope is the Price per game or how much were paying for 4 games

The student correctly calculates the slope of the line using values from the table and correctly interprets the slopes as the unit price.

c. Explain in words how you can use the slope to solve Mo'Nique's problem: How many computer games will cost $575?

I can make an equation so it would be Y= 23x than I can guese and check so I know it would have to be more than 16 games so I will see numbers between 16 and 30 so I tried 25 and it looked like Y= 23·25 And got $575 so 25 games cost 575

The student finds the correct equation for the line, $y=23x$ (where y represents the total price and x represents the number of games purchased). The student also correctly identifies 25 as the number of games

*(I can make an equation so it would be Y=23x than I can guese and check so I know it would have to be more than 16 games so I will see numbers between 16 and 30 so I tried 25 and it looked like Y=23*25 And got $575 so 25 games cost 575)*

This work represents a score point of 3. While the points are plotted incorrectly, this represents a minor error given that the work and reasoning shown reflect an accurate and clear understanding of the concepts and skills identified in the standards addressed by the task.

Student Work: Score Point 2

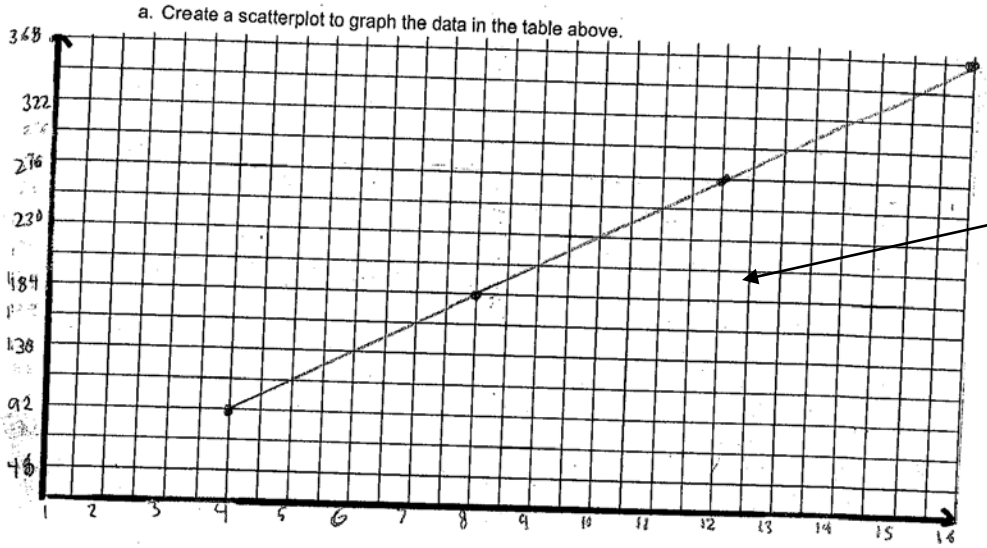
institute for . . lear

Mo'Nique discovered that four computer games cost $92. She built the following table to help her think through a problem, but didn't know what to do next.

23

Games	Cost
4	92
8	184
12	276
16	368

a. Create a scatterplot to graph the data in the table above.



The student plots points consistent with the identified scale; however, the scale is not consistent. The first box represents $46; those that follow represent $23.

b. Find the slope of the line that passes through the scatterplot, and explain in words what the slope means in the context of the problem.

92 ÷ 4 = 23 the slope means the cost of 1 game in this problem.

The student correctly calculates the slope of the line using values from the table and correctly interprets the slopes as the price of one game.

c. Explain in words how you can use the slope to solve Mo'Nique's problem: How many computer games will cost $575?

You can continue Following the pattern until you get to 575 dollars

The student identifies a method that could be used to find the number of computer games that will cost $575 but does not use slope or find the number of games.

This work represents a score point of 2. There are minor errors in the scale, and the student does not clearly demonstrate how the slope can be used to find the number of games that cost $575.

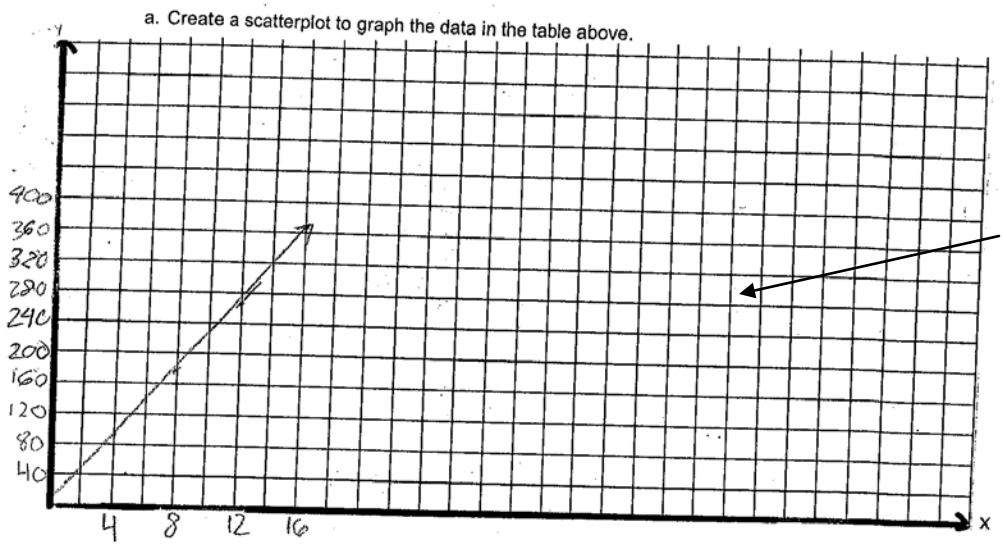
Student Work: Score Point 1

institute for

Mo'Nique discovered that four computer games cost $92. She built the following table to help her think through a problem, but didn't know what to do next.

Games	Cost
4	92
8	184
12	276
16	368

a. Create a scatterplot to graph the data in the table above.



The student plots points consistent with the identified scale.

b. Find the slope of the line that passes through the scatterplot, and explain in words what the slope means in the context of the problem.

The slope mean that for every four games, it is another 92 dollers. So, the slope stays constant.

The student provides a definition of the slope in the context of the problem, but does not use the provided information to calculate the slope.

c. Explain in words how you can use the slope to solve Mo'Nique's problem: How many computer games will cost $575?

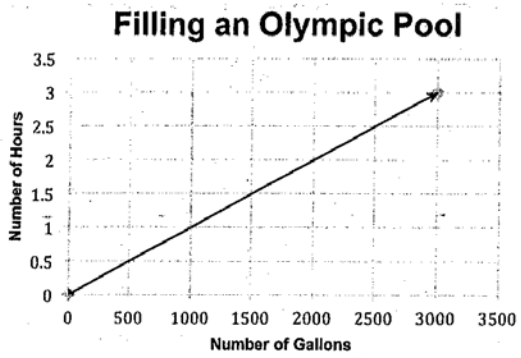
The amount of games that would cost $575 is 25.

The student provides a correct answer for the number of games that would cost $575 but does not show work or provide a written explanation of the process used to calculate it.

This work represents a score point of 1. The student does not provide the required response to part *b* (does not use the values to calculate slope) or part *c* (does not explain how the slope can be used to solve Mo'Nique's problem).

Student Work: Score Point 3

In the graph below, an Olympic-sized swimming pool is being filled with an average-sized garden hose.



a. At what hourly rate is the pool in the graph above being filled? Use mathematical reasoning to justify your response.

1000 gallons per hour because the line passes the point that says 1000 gallons and 1 hour at the same time.

b. Is the hourly rate at which the pool is filling the same as the slope of the line? Why or why not? Explain your reasoning in words.

yes because, the pool has 1000 more gallons after every hour. So the slope is $\frac{1}{1000}$.

c. Determine the equation of the line in the graph above. Show how you determined your answer.

$G=1000h$ I determined my answer by looking at the graph and discovering that it takes 1 hour to fill 1000 gallons so if you want to find out how many gallons will be filled in 5 hours you substitute 5 for h and multiply by 1000.

d. How long will it take to fill the 660,000-gallon pool? Use mathematical reasoning to justify your response.

It will take 660 hours because I set up a proportion, cross multiplied, and got 660.

$\frac{1}{1000} \frac{660}{660,000}$

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The student uses data from the graph to identify the hourly rate at which the pool is being filled.

While the student incorrectly answers the question in the affirmative, s/he correctly determines the slope of the line.

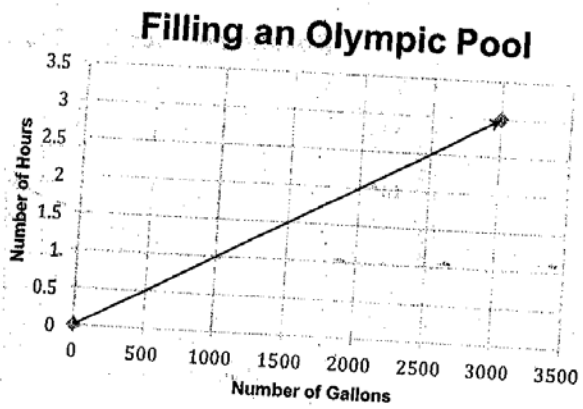
The student determines an equation that correctly represents gallons dependent upon the number of hours, in *x= (1/m) y* rather than *y= mx* form. This does not represent a major misunderstanding, given the student's correct calculation of the slope.

The student uses a proportion to correctly solve for the length of time required to fill a 660,000-gallon pool.

This work represents a score point of 3. The student correctly answers parts a, b, and d of the problem. In part c, the student provides a correct equation but does not express it in the form *y=mx*.

Student Work: Score Point 2

In the graph below, an Olympic-sized swimming pool is being filled with an average-sized garden hose.



a. At what hourly rate is the pool in the graph above being filled? Use mathematical reasoning to justify your response.

1000 gallon per hour

b. Is the hourly rate at which the pool is filling the same as the slope of the line? Why or why not? Explain your reasoning in words.

it was because the Number of gallons is more than so as if you do rise over run it will become a decimal.

c. Determine the equation of the line in the graph above. Show how you determined your answer.

$\frac{\text{Rise}}{\text{Run}} = \frac{1}{1000} = 0.001$

d. How long will it take to fill the 660,000-gallon pool? Use mathematical reasoning to justify your response.

it will take 660 hours I know because since 1 hour equals 1000 gallons I just keep counting

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The student correctly identifies the rate at which the pool is being filled.

The student correctly identifies the slope in part *c* and indicates correctly that the slope will be a decimal when you calculate it using the "rise over run" method.

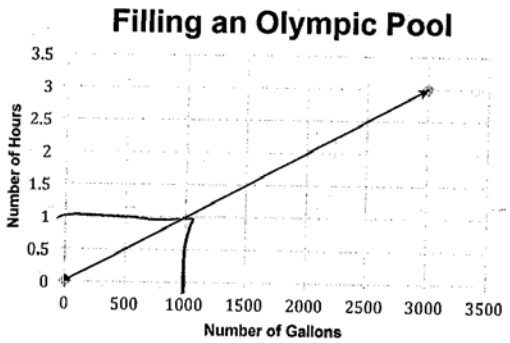
The student provides the correct slope but does not include the equation of the line.

The student correctly identifies the number of hours required to fill a 660,000-gallon pool by using a pattern.

This work represents a score point of 2. The student correctly answers parts *a*, *b*, and *d* of the problem. In doing so, the student demonstrates the ability to calculate the slope of a graph, identify an hourly rate, and distinguish between the rate in gallons per hour and hours per gallon. In part *c*, the student provides the slope of the line but does not provide the equation of the line.

Student Work: Score Point 2

In the graph below, an Olympic-sized swimming pool is being filled with an average-sized garden hose.



a. At what hourly rate is the pool in the graph above being filled? Use mathematical reasoning to justify your response.

The rate per hour is 1000 gallons because if you look at 1 hour it corects with 1000 gallons

b. Is the hourly rate at which the pool is filling the same as the slope of the line? Why or why not? Explain your reasoning in words.

Yes because the pool hour is going up by 0.5 and the gallons is going up by 500. 0.5 and 500 are proportional.

c. Determine the equation of the line in the graph above. Show how you determined your answer.

h = 1000G

d. How long will it take to fill the 660,000-gallon pool? Use mathematical reasoning to justify your response.

$\frac{h}{G} = \frac{1}{1000} = \frac{x}{660,000}$

it will take 660 hours or 27.5 days to fill the pool up.

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The student correctly identifies the rate at which the pool is being filled based on a point on the graph (1,000, 1).

The student correctly identifies this as a proportional relationship, but incorrectly answers "Yes." S/he doesn't explain the difference between the rate at which the pool is filling and the slope.

The student writes the correct equation for the line but does not show how the answer was determined.

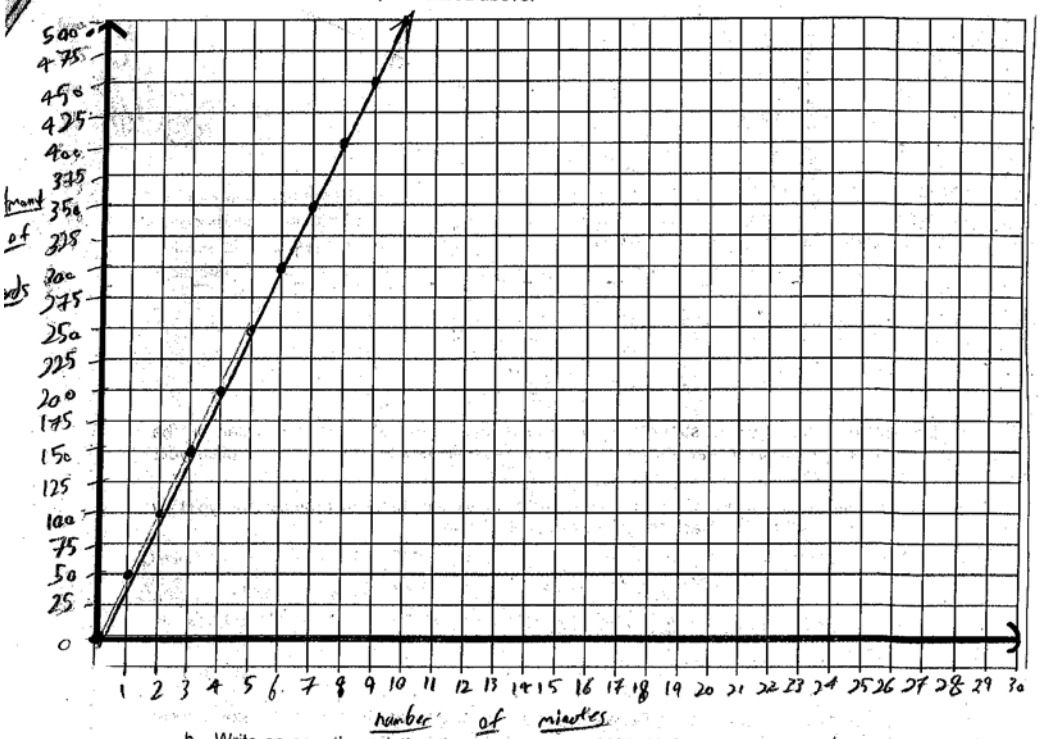
This work represents a score point of 2. The student correctly answers parts *a*, *c*, and *d* of the problem. Part *b* is answered incorrectly, as the student does not distinguish the rates in hours per gallon and gallons per hours. In part *c*, the student does not show how the equation for the line in the graph was determined.

The student sets and solves a proportion to find the correct amount of time required to fill the pool and converts the time in hours to time in days (660 hours = 27.5 days).

Score Point 2 (Sample A)

A professional typist can typically type 50 words per minute.

a. Graph the relationship described above.



The student graphs the relationship, with an appropriate and correct scale and correctly plotted points.

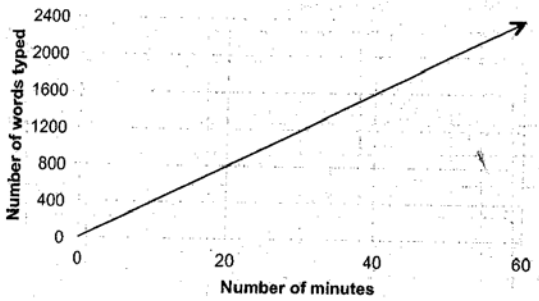
b. Write an equation relating the number of words typed (w) in terms of the time (t) in minutes. What information do the numbers in the equation represent? Explain your reasoning in words.

$t = 50w$

The number 50 means that a professional typist can type 50 words for every minute.

The student writes the correct equation for the relationship between words typed and time in minutes. S/he correctly identifies 50 as the number of words typed each minute.

c. The graph below shows the number of words typed by Donald over a 60-minute period of time. Write an equation relating the number of words typed in terms of the time in minutes. Use mathematical reasoning to justify your response.



$t = 40w$

The student writes the correct equation to express the number of words typed by Donald each minute but does not provide reasoning or rationale.

d. Determine if the following statements are true or false. Provide evidence based on the context of the problem that shows why the statements are true or false. Explain your reasoning in words.

i) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's graph to the professional's graph.

False because the numbers on both graphs don't increase at the same rate.

The student incorrectly writes that comparing the graphs does not help a person decide who's faster. The student correctly responds to part *ii*, recognizing the difference between the rates at which Donald and the professional typist can type.

ii) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's equation to the professional's equation.

True because, the professional typist types 10 words more than Donald in a minute. So after every minute the professional typist will be 10 words ahead than Donald.

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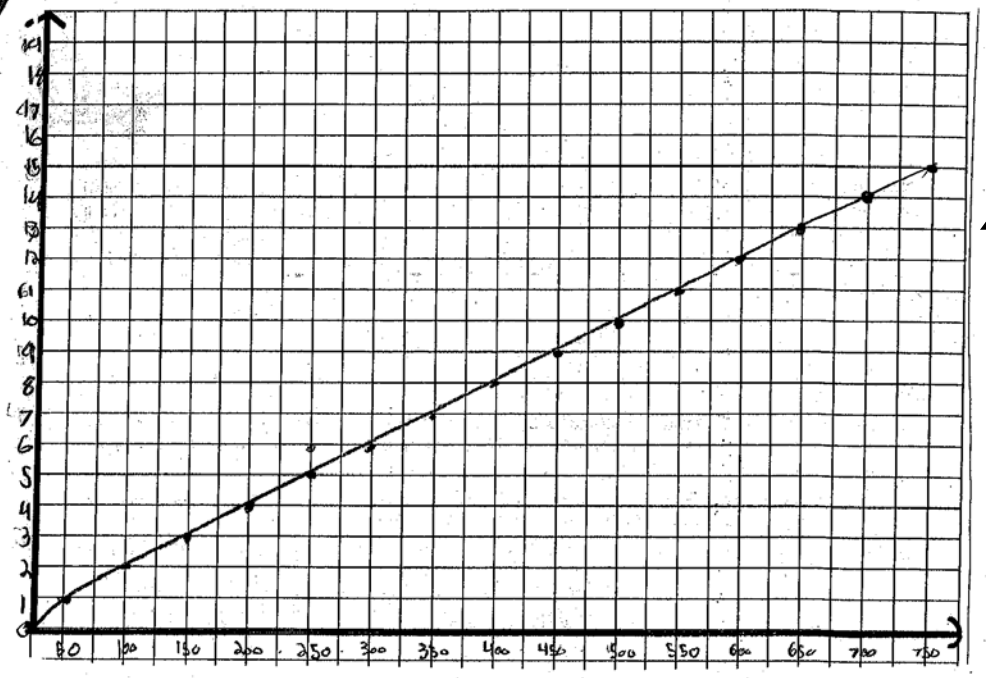
This work represents a score point of 2. The student is able to correctly graph the relationship in part *a* and write equations for the relationships in parts *b* and *c*. However, the student incorrectly answer part *d*, subpart *i*. The student does provide a correct explanation for part *d*, subpart *ii*, and recognizes that the professional typist types 10 more words per minute than Donald.

Score Point 2 (Sample B)

A professional typist can typically type 50 words per minute.

a. Graph the relationship described above.



b. Write an equation relating the number of words typed (w) in terms of the time (t) in minutes. What information do the numbers in the equation represent? Explain your reasoning in words.

Y = 50T

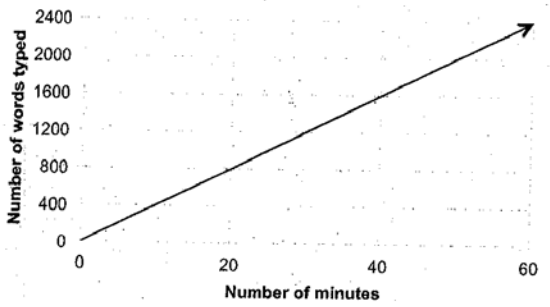
50 equals words per minutes

and T equals the time as in minutes.

The student includes an inconsistent scale on the *x*-axis from 0 to 50 and 50 to 100 words. The student graphs times on the *y*-axis and words on the *x*-axis. This representation is inconsistent with the equation included in part *b* (*Y=50T*); however, this representation can answer the question, "How long does it take to type *X* minutes?"

The student writes the correct equation and correctly explains the meaning of the number 50 and the variable *T*.

c. The graph below shows the number of words typed by Donald over a 60-minute period of time. Write an equation relating the number of words typed in terms of the time in minutes. Use mathematical reasoning to justify your response.



Y= 40x

The student writes the correct equation to express the number of words typed by Donald each minute but does not provide reasoning or rationale.

d. Determine if the following statements are true or false. Provide evidence based on the context of the problem that shows why the statements are true or false. Explain your reasoning in words.

i) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's graph to the professional's graph.

True but the graph would need to have the same numbers on the X or Y

The student answers correctly but provides a limited explanation. While the student alludes to the need for the same scales if the relationships are graphed on different axes, the language used is imprecise.

ii) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's equation to the professional's equation.

True because just by seeing that one equation has a bigger number than the other will show one types faster than the other (must be proportional)

The student answers correctly but provides a limited/unclear explanation. While s/he refers to the "bigger number" in one of the equations, s/he doesn't use precise language, e.g., coefficient.

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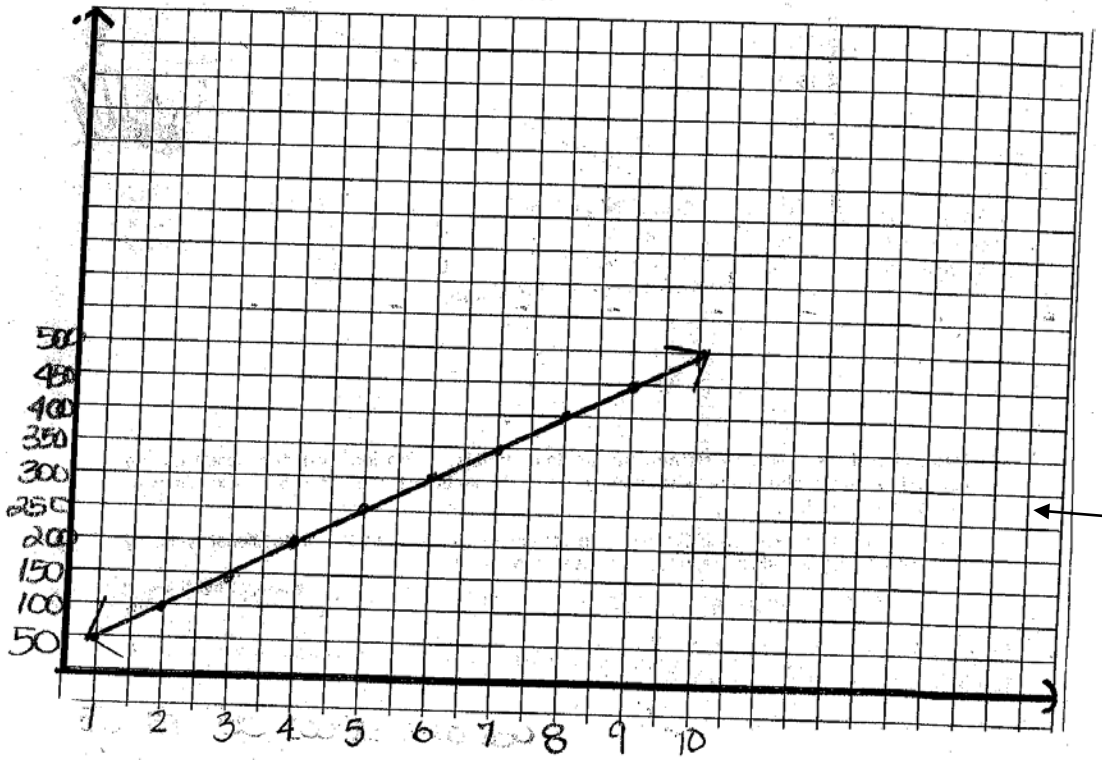
This work represents a score point of 2. The student graphs a relationship between words typed and time spent typing. The minor error in the graph scale does not reflect a conceptual misunderstanding. However, the representation in the graph and the equation are not consistent. The student correctly identifies the equation for the number of words typed by Donald but does not provide an explanation. While the student provides correct responses in part *d*, the explanations do not provide a sufficient argument or use appropriately precise language (e.g., coefficient, scale, etc.).

Score Point 1 (Sample C)

A professional typist can typically type 50 words per minute.

a. Graph the relationship described above.



The student graphs points consistent with the relationship provided. There is a minor error in the graph scale.

b. Write an equation relating the number of words typed (w) in terms of the time (t) in minutes. What information do the numbers in the equation represent? Explain your reasoning in words.

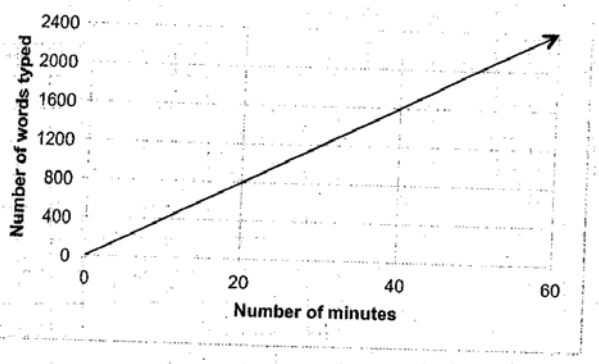
T	W
1	50
2	100
3	150
4	200

$y = 50x$

words Time

The student correctly finds the equation for the relationship between words typed and time. The student provides a table of values for the first 4 minutes of typing. The student identifies the quantities that the coefficient 50 and the variable *x* represent, but s/he does not elaborate further.

c. The graph below shows the number of words typed by Donald over a 60-minute period of time. Write an equation relating the number of words typed in terms of the time in minutes. Use mathematical reasoning to justify your response.



$y = 20x$

d. Determine if the following statements are true or false. Provide evidence based on the context of the problem that shows why the statements are true or false. Explain your reasoning in words.

i) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's graph to the professional's graph.

True because they can both show the number of words typed over a certain amount of time.

ii) A person can decide if Donald types faster or slower than the professional typist by comparing Donald's equation to the professional's equation.

False because they type a different amount of words in a certain amount of time so a table couldn't show acurate results compared to each other.

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The student finds an incorrect equation for the relationship between the number of words Donald typed and the amount of time spent typing. The student does not include a justification.

The student provides one correct response in part *d*. In subpart *i*, the student does not provide a sufficient explanation. In subpart *ii*, the student provides an incorrect explanation that reflects a limited understanding of the meaning of the coefficients in equations of proportional relationships.

This work represents a score point of 1. The student answers some parts of the problem correctly but provides limited explanations that do not sufficiently address the demands of the task. The explanations do not reflect an understanding of the connections among the equation, verbal descriptions, and graphs of a proportional relationship. Further, the student does not demonstrate performances indicative of the standards for mathematical practice. For example, the student does not use precise language in her/his written responses, nor does s/he provide justifications that meet the standard of argument expected of eighth graders.

GRADE 8 MATH: REPRESENTING AND INTERPRETING PROPORTIONAL RELATIONSHIPS

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline – Grade 8 Mathematics

Grade 8 Mathematics: Representing and Interpreting Proportional Relationships

Unit Overview

This unit builds directly on prior work in proportional reasoning and the use of expressions and equations in the 6th and 7th grades and extends the ideas more formally into the realm of algebra. Proportional relationships provide an entry point for students to engage more formally with expressions and equations and the relationships they represent (modeled in graphical and tabular form, as well). This unit continues to deepen students' abilities to model real-world mathematical problems with graphs, tables, and equations, to understand the connections among these models, and to use these models to better understand problems.

Prior to engaging with this unit, students should be familiar with the use of variables to represent numbers and unknown quantities. In the 6th and 7th grades, students should have had numerous opportunities to translate word problems and verbal descriptions into algebraic expressions and equations with variables. Students should be able to confidently evaluate expressions and equations and use the properties of operations (e.g., the distributive, commutative, and associative properties) to generate equivalent expressions. In 7th grade, students will have done significant work with proportional relationships, leading to understandings of several key concepts, including the constant of proportionality and representations of proportional relationships with graphs and equations. Students should be able to use rates, ratios, and proportional relationships to solve a variety of problem types before engaging in this unit.

Following this unit, students should continue their studies of expressions, equations, and functions. Deeper study should be devoted to pairs of simultaneous linear equations, briefly addressed in this unit. Further study should include opportunities to develop fluency in solving systems of two linear equations and applying that fluency in problem-solving contexts. Additionally, a more rigorous treatment of functions, including the understanding of domain and range and properties of functions, should be developed.

A description of how these concepts are developed can be found in the *Progression for the Common Core State Standards in Mathematics* (draft) April 22, 2011 (http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf).

Unit Outline – Grade 8 Mathematics

Unit Topic and Length

The sequence of related lessons is intended to take 25-35 instructional periods (45-minutes), depending on students' prior knowledge and work with proportional relationships, expressions, and equations. It is recommended that this unit be taught during the first quarter of the school year.

Common Core Learning Standards

Standards for Mathematical Content:

- **7.RP.2** Recognize and represent proportional relationships between quantities.
- **7.RP.2b** Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
- **8.EE.5** Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
- **8.EE.6** Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y=mx$ for a line through the origin and the equation $y=mx+b$ for a line intercepting the vertical axis at b.
- **8.F.4**. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

Standards for Mathematical Practice:

- **MP.1** Make sense of problems and persevere in solving them.
- **MP.2** Reason abstractly and quantitatively.
- **MP.3** Construct viable arguments and critique the reasoning of others.
- **MP.4** Model with mathematics.
- **MP.6** Attend to precision.
- **MP.7** Look for and make use of structure.
- **MP.8** Look for and express regularity in repeated reasoning.

Unit Outline – Grade 8 Mathematics

Big Ideas/Enduring Understandings	Essential Questions
➢ When two quantities vary in such a way that one of them is a constant multiple of the other, the two quantities are "proportional". Conversely, two quantities are proportional when they vary in such a way that one of them is a constant multiple of the other, i.e., they have a constant ratio. ➢ When two quantities, x and y, vary in such a way that one of them is a constant multiple of the other, a model for that situation is $y = kx$ where k is the constant of proportionality or the constant ratio of y to x. ➢ The slope of the graph of a proportional relationship is the constant of proportionality or the constant ratio of y to x. ➢ The equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b can be derived by using similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane. ➢ A linear function can be represented by an equation of the form $y = mx + b$ where m and b have a regular and predictable meaning in the context, table, graph, and equation. ➢ Slope and intercept can be interpreted within the context of a real-world problem. From *Institute for Learning Eighth Grade Unit Map: Expressions and Equations* © 2011 University of Pittsburgh	➢ How can patterns, relationships, and functions be used as tools to best describe and help explain real-life relationships? ➢ How can the same mathematical relationship be represented in different ways? Why would that be useful? ➢ How does the unit rate of a proportional relationship help me determine the slope of a graph? ➢ How can similar triangles be used to explain the concept of slope? ➢ How can the rate of change and initial value of a function be determined from a description of a relationship or from two (x, y) values?

Unit Outline – Grade 8 Mathematics

Content	Skills
➢ Linear relationships represented as tables of values, graphs, equations, and verbal descriptions/real world situations ➢ Slope-intercept form for linear equations ➢ Connections among representations of a relationship as an equation, graph, or table of values, and/or verbal descriptions/real world situations ➢ Slope as a rate of change ➢ Proportionality and direct variation	➢ **Graph** linear equations from a table of values. ➢ **Graph** linear equations using slope-intercept form. ➢ **Construct** a function to model a linear relationship between two quantities. ➢ **Interpret** graphs of linear equations. ➢ **Represent** linear relationships using tables, graphs, equations, and verbal descriptions/real world situations. ➢ **Compare** linear relationships based on slope/rate of change in tables, graphs, equations, and verbal descriptions/real world situations.

Key Terms/Vocabulary

Slope, proportionality, rate (of change), function, linear, equation, direct variation, intercept , similar, similar triangles, constant of proportionality, unit rate,

Assessment Evidence and Activities

Initial Assessment : Filling and Draining a Pool

In the *Filling a Pool* task, students are presented with a situation in which they must use the rate at which a pool is filled to determine how long it will take to fill. Students are expected to represent this data in both a table and a graph.

In the *Draining a Pool* task, students represent the situation of draining the pool with a graph, given a situation in which the rate and the starting volume are provided.

Both tasks provide evidence of student understanding of rates of change and the relationships among different representations of these two situations.

Learning Plan & Activities

➢ This unit contains a pre-assessment task, formative assignments, and a final assessment. The mathematics of this unit requires students to understand that functions can be represented in multiple ways and to communicate their interpretation of these representations. In addition, great emphasis is placed on recognizing slope as a rate of change as well as identifying proportionality in different function representations. Students will work with tables, graphs, equations, and situational representations of functions. There

is a focus on noticing patterns and making connections to how the patterns can be used to recognize linear functions.

- **Pre-Assessment**: This unit begins with 2 different versions of a pre-assessment called *Filling a Pool* and *Draining a Pool*. In these activities students are presented with information about the rates at which a pool can be filled or drained and the amount of water it contains. Students are asked to represent this information on a graph, determine its linear equation, and interpret the different terms within the equation. This builds on prior knowledge of using a table to graph a linear function from a table.
- It should be noted when looking at student work for the pre-assessment task, students may not understand the following: identifying the role of slope, understanding that when the pool is being drained there must be a negative slope, and realizing that the initial pool volume is the y-intercept.

- Throughout the unit, students complete **Mastery Checks**. Mastery Checks are quick formative assessments based on unit standards. Students' solutions are then assessed and their progress is tracked. If they show a strong understanding of the standard, it is noted that they have mastered the standard. If there is evidence of a procedural or conceptual misunderstanding, this is noted on the standard tracking sheet, allowing for targeted grouping to eliminate the misconception. This targeted meeting could consist of but is not limited to teacher conferences, small groupings, pull-out services, or individual student work.
- There should be strong emphasis on mathematical discourse in the classroom. Students should be held accountable for the use of precise vocabulary and meanings of numbers, terms, and variables and the quantities they represent. Time should be provided for students to investigate and discuss the numerous patterns that emerge when working with linear functions in multiple representations. Students must be able to understand and communicate what these patterns are and why they exist. This provides teachers with additional opportunities to assess student understanding.

Additional Supports

Clarify vocabulary and symbols by pre-teaching and re-teaching vocabulary and symbols, such as proportional relationship; unit rate; rate of change; constant of proportionality; mathematical reasoning; justify; ratio; unit rate; linear relationship; y-intercept; slope; linear equations; non-linear equations; and rise/run.

- As students discuss the patterns that emerge in proportional relationships, they should be provided with time (and prompting, if needed) to articulate the relationships between and among different solution pairs that satisfy the conditions of the function. A rich problem—with extended discussion and share time—provides opportunities for students to recognize, for example, that between any two solution pairs (x_1, y_1) and (x_2, y_2) that satisfy a proportional relationship, it follows that — — and — —.
- Questions should be asked to assess and advance students' understanding of the meaning of the numbers in the ratios they identify (e.g., What does x represent in

this ratio?), the value and meaning of slope, or *m* in linear equations (e.g., What does the slope in this equation/of this line mean? What is changing? Is the slope positive or negative and what does that mean in terms of the problem?), and proportionality (e.g., Why is this a proportional relationship? What is the relationship among solution pairs in a proportional relationship?).

- **Teacher as a facilitator**: Students do their best learning and sharing of ideas when the teacher can take on the role of a facilitator in the classroom. This means limiting back-and-forth questioning with the whole class or a small group and asking questions that allow students to think, make connections, and form ideas based on what they're observing.
- **Presentation of Work**: In order to explain proportionality of functions in different representations, provide students with examples in each form that fit into each category. This allows them to form their own understanding and explanations of what make something proportional. This also encourages communication and stronger understanding by the whole class as to what makes relationships proportional.
- **Reflections:** Students should be asked to explain their understanding in a written form on a regular basis. This includes but is not limited to short answer responses, explanations of concepts/new ideas (e.g., a letter to an absent classmate), or full page journals where they explain different ideas within the unit. This allows for additional communication grounded in deep conceptual understanding as wells as an additional opportunity to assess student understanding.
- **Progress Conferences**: Conference with students to make them aware of their progress relative to goals that have been set. This can be done individually, time permitting, or in small student groups. Tracking student performance relative to objectives and the standards allows for flexible grouping of students with similar strengths or misconceptions. As students are made aware of areas in which improvements in performance are necessary, they should set goals and be held accountable to the achievement of those goals.

RESOURCES

- Graph paper, calculators (scientific and graphing), and rulers
- Included in this bundle are the pre-assessment, formative assessments, and referenced activities in addition to the final assessment.
- *Math Snacks* (http://mathsnacks.com/) provides videos that explain 6th-8th grade mathematics content in animated stories. These brief videos can support students with background, concept development, and connections/reinforcement of previously learned content. The "snack" entitled *Overruled!* (http://mathsnacks.com/overruled.php) is particularly relevant for the content of this unit.
- For guidance on planning lessons around high-cognitive demand tasks, see
  - "Thinking through a Lesson: Successfully Implementing High-Level Tasks," Smith, Margaret S., Bill, Victoria, and Hughes, Elizabeth K. 2008. *Mathematics Teaching in the Middle School,* Vol. 14, No. 3.
  - "Orchestrating Discussions," Smith, Margaret S., Hughes, Elizabeth K., Engle, Randi A., and Stein, Mary Kay. 2009. *Mathematics Teaching in the Middle School,* Vol. 14, No. 9.

Unit Outline – Grade 8 Mathematics

CURRICULAR RESOURCES

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Lesson	Description/Unit Content Addressed
1.1: Direct Variation	In this lesson, a series of three investigations, students explore the differences between proportional and non-proportional relationships, are introduced to inverse linear relationships, and translate among various representations of relationships (including tables, graphs, written descriptions, and equations). 8.EE.5, 8.F.4
1.2: Slope	In this lesson, students are introduced to the concept of slope through a consideration of "steepness." Explicit connections are made to the slopes of lines. 8.EE.6
1.3: Write Equations	In this lesson, students are provided with a series of investigations in which they determine whether a relationship is linear, express and interpret equations in the form $y = ax + b$, explore the relationships among various forms in which a relationship can be expressed, and write equations for graphs of linear relationships. 8.F.4

Formative Assessment Tasks

<table>
<tr><th>Task</th><th>Implementation Notes</th></tr>
<tr><td>

Larry's Job

Larry makes $50 a week. Show this relationship in a graph below.
(*provide graph paper for student response*)

What is the slope of the line in the equation?

What is the connection between the slope of the line and the amount of money Larry makes per week?

Write the equation for this relationship.

Is this a proportional relationship? How do you know?

</td><td>

In this task, students represent one relationship graphically and as an equation. Explicit connections to slope provide opportunities for students to make connections among the numbers in the verbal description of Larry's job, the graph, and the equation.

Probe students further by asking what the slope means and how they found it. Also, consider providing students with a written response (here or later in the unit) in which they explain where slope is connected to both the graph and the equation of linear relationship.

Proportionality is revisited again. Students who have difficulty with this question should be supported intensively, as representations (in graphs, words, tables, and equations) of proportional relationships are a focus of the unit.

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Gamers

Gamers	**Game World**
Two dollars for every game download	$15 per month and $0.50 per game downloaded

1. How much money would it cost if you downloaded 6 games from **Gamers**? Show your work, using the equation *c=2d*, where *c* is the cost and *d* is the number of downloads.

2. Write an equation for the cost per month at **Game World**, using *c* for cost and *d* for downloads.

</td><td>

The problem sets up two situations, each of which can be modeled in a variety of ways. The relationship between cost and number of games at Gamers is proportional. The relationship at Game World is not.

In this example, students are asked to use and evaluate a provided equation. They should be encouraged to discuss how the variables *c* and *d* change in response to each other. E.g., How does the cost change as the number of downloads increases from 5 to 10? If the cost increases by 10, how many more games were downloaded?

In this question, students are expected to generate an equation for a non-proportional relationship. Students who are having difficulty with this problem should be encouraged to represent the relationship in

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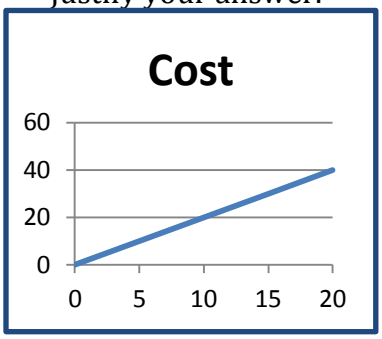
	multiple ways to help as they write the equation.
3. Which equation is represented in the graph below, Gamers or Game World? Justify your answer. 	Students should be able to compare ordered pairs on the line to the equations for the two game sites or use the properties of proportional relationships and their graphs/equations to determine which game site is graphed here. Students may reference *intercept* in their explanation.
4. If you plan on downloading 8 games per month, which would be the best plan to buy? Justify your answer using mathematical reasoning.	Students can compare graphs or input values into the equations to determine the cost for 8 monthly downloads from each site.
5. Justin has a membership to Gamers and he says his plan is better than Game World's. Kenny, who has a membership to Game World, disagrees. Who is correct? Justify your answer using mathematical reasoning.	Question 5 provides a good opportunity for partner or group sharing. Consider having students explain their reasoning orally once they've had an opportunity to express it in writing. The open-endedness of this question means that students must explain constraints themselves.
6. How many games would Justin and Kenny have to download to make the same monthly payment? How do you know?	In this question, students are being asked to solve a system of linear equations. Students have limited exposure/practice with formal algorithms for solving systems of equations, so alternative methods, including inspection of graphical representations, comparing a table of values, and guess-and-test strategies should be encouraged and validated.

INITIAL ASSESSMENT

Filling the Pool

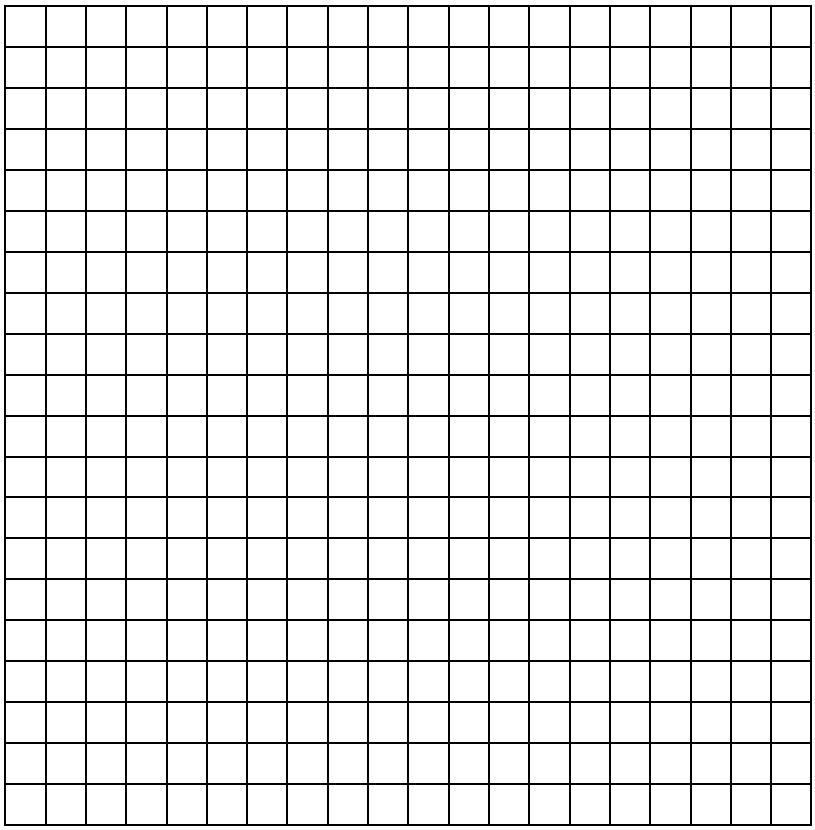
Summer is about to begin! You just got a job working in one of the local hotels, and you are in charge of pool maintenance. You are responsible for cleaning out and filling the pool this week.

- The pool holds a total of 12,000 gallons of water
- The pool fills at a rate of 1,400 gallons per hour

a. Complete the table to show how much time it will take to fill the pool entirely.

hours														
gallons														

b. Graph time in hours versus the amount of water in the pool in gallons.



c. Determine whether the relationship is proportional. Justify your answer with mathematical reasoning.

d. Find the equation of the line, and explain the meaning of slope in this problem.

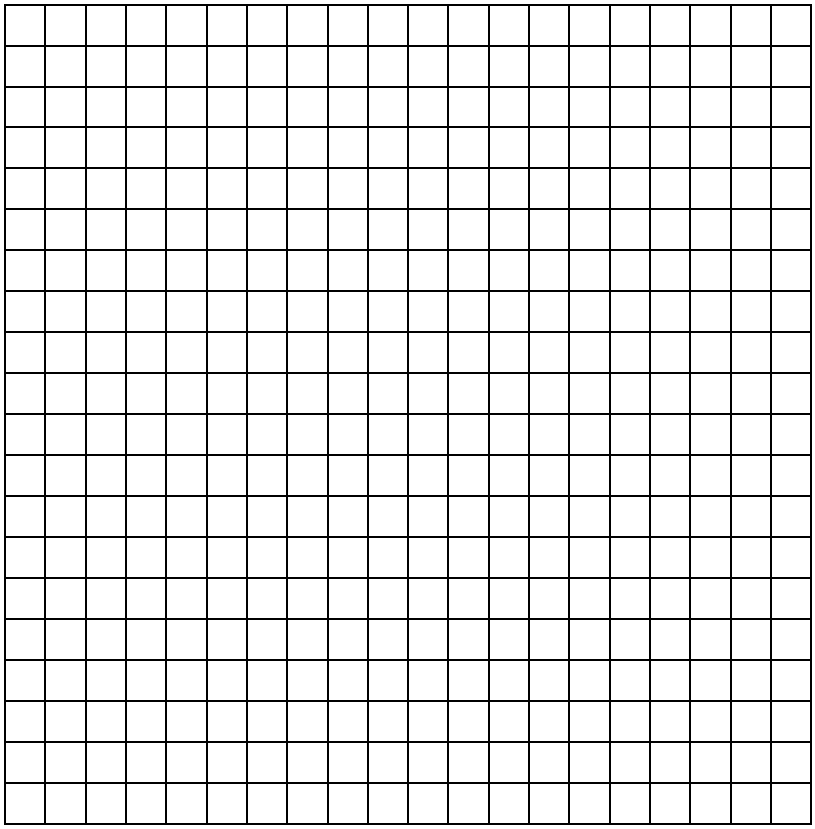
e. What would happen if 1,000 gallons of water was already in the pool when you began filling it? Graph one the coordinate grid above, and find the equation.

Draining the Pool

Summer is officially over. Not only are you back in school, but it's time to help clean out the swimming pool. Your first responsibility is to drain water from the pool.

- The pool holds 12,000 gallons of water
- The pool drains at a rate of 1,400 gallons per hour

a. Graph time in hours versus the amount of water left in the pool in gallons, and determine whether the relationship is proportional. Explain how you made your decision.



b. Find the equation of the line and explain the meaning of the slope in this problem.

c. Is this relationship proportional? Justify your answer using mathematical reasoning.

d. Using the same grid above, graph the line of a 9,000 gallon tank that drains at the rate of 1,400 gallons per hour. Write the equation for the line.

e. Compare the graphs and explain any similarities or differences you observe.

Standard	Remarks
7.RP.2a, c, d	
8.EE.5	

SAMPLE MASTERY CHECK

Interpreting the Equation of a Line
(7.RP.2a, c, d; 8.EE.5)

1. The equation of a line can be written as $y = mx + b$.

   Explain what each letter in this equation stands for

Letter	What does it stand for?
y	
m	
x	
b	

2. Write a linear equation using x and y as variables.

3. Construct a table of values using your equation.

x	y

4. How did the y-value change as the x-value increased by 1?

   Why did this occur? What part of your equation represents that change?

5. Describe what this relationship would look like if it were in graphical form.

6. Is this a proportional relationship? Justify your answer using mathematical reasoning.

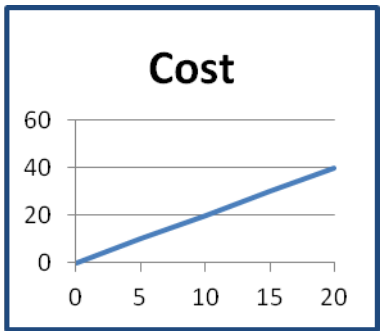
FORMATIVE ASSESSMENT: GAMERS

Gamers	**Game World**
Two dollars for every game download	$15 per month and $0.50 per game downloaded

1. How much money would it cost if you downloaded 6 games from **Gamers**? Show your work, using the equation $c=2d$, where c is the cost and d is the number of downloads.

2. Write an equation for the cost per month at **Game World**, using c for cost and d for downloads.

3. Which equation is represented in the graph below, Gamers or Game World? Justify your answer.



4. If you plan on downloading 8 games per month, which would be the best plan to buy? Justify your answer using mathematical reasoning.

5. Justin has a membership to Gamers and he says his plan is better than Game World's. Kenny, who has a membership to Game World, disagrees. Who is correct? Justify your answer using mathematical reasoning.

6. How many games would Justin and Kenny have to download to make the same monthly payment? How do you know?

FORMATIVE ASSESSMENT: LARRY'S JOB

Larry makes $50 a week. Show this relationship in a graph below.
(*provide graph paper for student response*)

What is the slope of the line in the equation?

What is the connection between the slope of the line and the amount of money Larry makes per week?

Write the equation for this relationship.

Is this a proportional relationship? How do you know?