SCIENCE | TECHNOLOGY | ENGINEERING | MATHEMATICS
STEM

NEW YORK CITY DEPARTMENT OF EDUCATION
DIVISION OF TEACHING AND LEARNING
OFFICE OF CURRICULUM, INSTRUCTION & PROFESSIONAL LEARNING

From Answer Finding to *Doing*

6–12 Mathematics Breakout Session

Introduction to Professional Learning for New Teachers

August 25–26, 2015

AGENDA

- Introductions & Norms
- Hexagon Task Lesson: Tables & Chairs
- Hexagon Task Lesson: Planning Highlights
- Engaging, Exploring & Sharing: Shifts in Mathematics
- Case Study: Teacher Actions that Support Doing Mathematics
- Closing & Reflection

SESSION OUTCOMES

Participants will:

- Engage in a model lesson centering on a mathematics task that is intellectually engaging and enjoyable and analyze pedagogical techniques that support student learning.
- Experience and analyze a mathematics lesson structure that promotes sense making and students as doers of mathematics.
- Become familiar with the eight Mathematics Teaching Practices from NCTM and use them to analyze a case study of a classroom engaging with a mathematics task.
- Gain insight into a lesson structure and pedagogical practices that promote deeper mathematical learning and discuss the impact on lesson planning.

NYC Department of Education
Carmen Fariña, Chancellor

Effective Collaboration Norms and Guidelines

In order to cultivate a climate where everyone is focused on ongoing, positive growth and improving student achievement, use the Seven Norms of Collaboration.

Seven Norms of Collaboration

1. **Promoting a Spirit of Inquiry and Balancing Advocacy**
   Exploring perceptions, assumptions, beliefs, and interpretations promotes the development of understanding. Inquiring into the ideas of others before advocating for one's own ideas is important to productive dialogue and discussion. Advocacy after thoughtful inquiry moves us towards decision-making. Groups take care to balance advocacy with inquiry, so as not to rush to decision-making nor leave issues without closure.

2. **Pausing**
   Pausing before responding or asking a question allows time for thinking and enhances dialogue, discussion, and decision-making.

3. **Paraphrasing**
   Paraphrasing is a powerful way to indicate that you are listening to others and are trying to understand them. Maintain the intention and accuracy of what the speaker said as you recast their contribution in your own words or with an example. Using a paraphrase starter that is comfortable for you – "So…" or "As you are saying…" or "You're thinking…" – and following the starter with an efficient paraphrase assists members of the group in hearing and understanding one another as they converse and make decisions.

4. **Probing**
   Using gentle open-ended probes or inquiries – "Please say more about…" or "I'm interested in…" or "I'd like to hear more about…" or "Then you are saying…" – increases the clarity and precision of the group's thinking.

5. **Putting Ideas on the Table**
   Ideas are the heart of meaningful dialogue and discussion. Label the intention of your comments. For example: "Here is one idea…" or "One thought I have is…" or "Here is a possible approach…" or "Another consideration might be…."

6. **Paying Attention to Self and Others**
   Meaningful dialogue and discussion are facilitated when each group member is conscious of self and of others, is aware of what (s)he is saying, and how it is said as well as how others are responding. This includes paying attention to learning styles when planning, facilitating, and participating in group meetings and conversations.

7. **Presuming Positive Intentions**
   Assuming that others' intentions are positive promotes and facilitates meaningful dialogue and discussion, and prevents unintentional put-downs. Using positive intentions in speech is one manifestation of this norm.

Adapted from CCE and the work of R. Garmston – November 2010

Name ..... Date ..... Class .....

Tables & Chairs

The diagram below shows the first four arrangements of hexagon-shaped banquet tables. The first arrangement consists of a single banquet table, and in each subsequent table arrangement, one additional hexagon-shaped table is added on to the end.

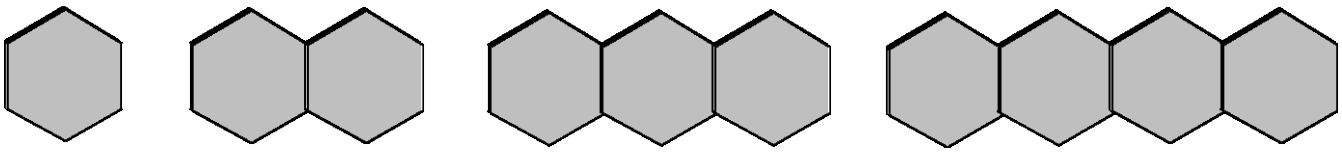

Table Arrangement 1 *Table Arrangement 2* *Table Arrangement 3* *Table Arrangement 4*

If one chair can be set up along each edge of a hexagon table, write an equation that relates the total number of chairs to any table arrangement.

Tables & Chairs: ***Hexagon Task Lesson***

Reflection Tool

1. What **stands out to you most** about this classroom experience?

2. What **teaching moves** do you feel were effective in promoting student learning and understanding?

3. What **features of the task** itself helped to promote discussion?

Thoughts from Table Talk:

Characteristics of Mathematical Tasks at Four Levels of Cognitive Demand

From Smith and Stein, 1998, as displayed in *Principles to Actions,* NCTM, 2014 (page 18).

Levels of Demands

Lower-level demands (memorization):

- Involve either reproducing previously learned facts, rules, formulas, or definitions or committing facts, rules, formulas or definitions to memory
- Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure
- Are not ambiguous. Such tasks involve the exact reproduction of previously seen material, and what is to be reproduced is clearly and directly stated.
- Have no connection to the concepts or meaning that underlie the facts, rules, formulas, or definitions being learned or reproduced

Lower-level demands (procedures without connections):

- Are algorithmic. Use of the procedure either is specifically called for or is evident from prior instruction, experience, or placement of the task.
- Require limited cognitive demand for successful completion. Little ambiguity exists about what needs to be done and how to do it.
- Have no connection to the concepts or meaning that underlie the procedure being used
- Are focused on producing correct answers instead of on developing mathematical understanding
- Require no explanations or explanations that focus solely on describing the procedure that was used

Higher-level demands (procedures with connections):

- Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas
- Suggest explicitly or implicitly pathways to follow that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts
- Usually are represented in multiple ways, such as visual diagrams, manipulatives, symbols, and problem situations. Making connections among multiple representations helps develop meaning.
- Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with conceptual ideas that underlie the procedures to complete the task successfully and that develop understanding.

Higher-level demands (doing mathematics):

- Require complex and nonalgorithmic thinking—a predictable, well-rehearsed approach or pathway is not explicitly suggested by the task, task instructions, or a worked-out example.
- Require students to explore and understand the nature of mathematical concepts, processes, or relationships
- Demand self-monitoring or self-regulation of one's own cognitive processes
- Require students to access relevant knowledge and experiences and make appropriate use of them in working through the task
- Require students to analyze the task and actively examine task constraints that may limit possible solution strategies and solutions
- Require considerable cognitive effort and may involve some level of anxiety for the student because of the unpredictable nature of the solution process required

These characteristics are derived from the work of Doyle on academic tasks (1988) and Resnick on high-level-thinking skills (1987), the *Professional Standards for Teaching Mathematics* (NCTM 1991), and the examination and categorization of hundreds of tasks used in QUASAR classrooms (Stein, Grover, and Henningsen 1996; Stein, Lane, and Silver 1996).

CCSS Mathematics » Standards for Mathematical Practice

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to *contextualize*, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.

3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$, and $(x - 1)(x^3 + x2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

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Lesson Structure for *Doing* Mathematics

Key Features:

- Sense-making and assessment happen throughout
- Relatable, but not necessarily real world
- Focusing, not funneling

Engage
Whole Class

- Develop enthusiasm
- Access prior knowledge
- Clarify conditions

Explore
Small Group

- May begin independently
- Develop solution path
- Articulate thinking

Share
Whole Class

- Share and connect solution paths
- Identify and analyze errors
- Concretize learning

Teacher Role:

- Make student thinking explicit
- Connect student thinking

Student Role:

- Participate in a community of mathematicians

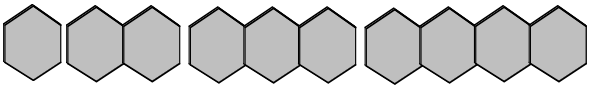
During the Hexagon Task lesson, what were the teacher and students doing during each phase?

ENGAGE	
Teacher Actions	**Student Actions**
EXPLORE	
Teacher Actions	**Student Actions**
SHARE	
Teacher Actions	**Student Actions**

Exploring Linear Functions: *The Case of Ms. Peterson*[1]

Ms. Peterson wanted her eighth grade students to understand three key ideas about linear functions: 1) variables can be used to represent two quantities that change in the relationship to each other; 2) that there are different but equivalent ways of writing an explicit rule; and 3) that connections can be made between different representational forms – tables, graphs, equations, words, and pictures. She selected the Hexagon Pattern Train Task because it aligned with her goals for the lesson and with state standards, it had multiply entry points and solution paths, and it would challenge her students to think and reason.

Trains 1, 2, 3, and 4 are the first 4 trains in the hexagon pattern. The first train in this pattern consists of one regular hexagon. For each subsequent train, one additional hexagon is added.



train 1 train 2 train 3 train 4

1. Compute the perimeter for each of the first four trains;
2. Draw the fifth train and compute the perimeter of the train;
3. Determine the perimeter of the 25th train without constructing it;
4. Write a description that could be used to compute the perimeter of any train in the pattern; and
5. Determine which train has a perimeter of 110.

Ms. Peterson told students that questions 1-3 were to help them get started if they needed it, question 5 was for students who finished quickly, but that question 4 would be the focus of the whole class discussion. She reminded students that they could use any of materials (hexagon shapes, copies of the hexagon trains, colored pencils, calculators) that were available on their tables.

As students began working in their groups, Ms. Peterson walked around the room stopping at different groups to listen in on their conversations and to ask questions as needed. When groups struggled to get started, she asked: "What would train 5 look like? How much bigger is the perimeter of train five than the perimeter of train 4? What is different as you move from one train to the next? How does the pattern seem to be growing?" She also encouraged students to use the hexagon shapes to build new trains and to make observations about what changes and what remains the same as the trains get bigger. When groups determined a correct algebraic representation she pressed them to explain what each part of the equation meant, to relate each part of the equation to the picture, and to explain how they knew it would always work.

Ms. Peterson spent more time with Groups 2 and 5. Group 2 had identified the growth rate as +4 but was not able to find the perimeter of a train without knowing the perimeter of the train that preceded it. She asked: "Why are you adding 4? Where is the 4 in the picture? Can you predict the number of 4's that you are adding each time?'' Group 5 had reasoned that since a hexagon has 6 sides that every time you add one more hexagon you add 6 more sides. She asked: "When you use your rule to find the perimeter in train 4, do you get the same answer as you did when you counted? Why not? What gets counted when you multiply by 6?"

As she visited the groups, Ms. Peterson made note of the strategies students were using (see reverse side) so she could decide which groups she wanted to have present their work. Each presenting group would be expected to explain what they did and why, how their equation connected to the visual arrangement of hexagons, and to answer questions posed by their peers. Group 3 would go first since their table would be accessible to all groups. Group 1 would go next because the equation they created modeled the verbal description given by Group 3 but was derived from the picture rather than from numbers in a table. Group 5 would then present a different equation, also derived from the picture, and discuss how they came to see that

[1]This case, written by Margaret Smith (University of Pittsburgh), was inspired by lessons planned and taught by Timothy Booth, a graduate student at the University of Pittsburgh and other teachers with whom she has worked over the last decade. It is intended to support the Teaching and Learning Guiding Principle in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM 2014).

counting all of the sides did not work. The presentations would conclude with Group 2. Below is an excerpt from the discussion that took place around the explanation presented by Group 2.

Michael: If you look at Train 4 we saw that there were five sides on each end of the train that counted in the perimeter and that the two hexagons in the middle each had 4 sides that counted. So we had 5 + 4 + 4 + 5 which is 18.

Ms. P.: So suppose we were talking about the 10th train. Can someone else explain how we could use Group 2's method to find the perimeter of the 10th train? *(Several students raise their hands and the teacher calls on Kelsey.)*

Kelsey: For Train 10 it would 5 + 5 for the two ends and then there would be 8 4's in the middle. So it would be 32 + 10 which is 42.

Brian: How did you know how many 4's would be needed?

Kelsey: I pictured it in my head and if I took off the two ends then there would be 8 hexagons left.

Ms. P.: I am wondering if we could generalize the number of hexagons that are between the two ends of any train. Take two minutes and turn and talk to the person sitting next to you. *(After two minutes the teacher continues.)* Amber? What did you and Sara decide?

Amber: Well we think that the number of hexagons in the middle is h – 2. Then you would need to multiply by 4 to get the total number of sides that the ones in the middle contribute to the perimeter. That is 4(n-2).

Ms. P.: What do others think? *(Students are nodding their heads or giving a "thumbs up".)*

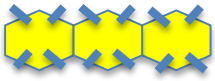
Ms. P.: If we use 4(n-2) like Amber and Jackie are suggesting, will we get the correct perimeter?

Sara: No. You need to add on the 5 for the first hexagon and the 5 for the last hexagon. So it has to be P= 4(n-2) + 10.

Ss: YES!

With only five minutes left in the class, Ms. Peterson asked students to picture in their minds what the graph of this function they had been working on would look like, to sketch the graph, and explain why they think their sketch made sense. She would use this information to see what students understood about relationship between representations and to launch class the following day. She also planned to talk about how there could be three (or more) equations for the same function.

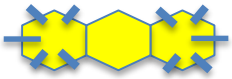
Group 1's Solution



Each hex has 2 sides on the top and 2 on the bottom that count. So that's 4. Then you need to add the two on the ends.

It is P = 4h +2

Group 2's Solution



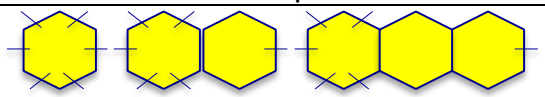
There are five sides on each end of the train. Then every time you add another hexagon there are 4 more sides on the inside – 2 on the top and 2 on the bottom.

Group 3's Solution

Train #	Perimeter
1	6
2	10
3	14
4	18
5	22
6	26
7	30
8	34

The perimeter of each train is 4 more. So if you multiply the number of hexagons by 4 you need to add on 2 to get the right perimeter.

Group 4 Solution



The first hexagon train has six sides. When you go to bigger trains the right side of the first hexagon moves to the end of the train. Each of the other hexagons in a train adds 4 sides to the perimeter.

P = 4(h – 1) + 6

Group 5's Solution

Each hexagon has six sides, but all of them don't count. So you have to get rid of the vertical ones. So that would be 2x. But you then have to add two back for the sides on the ends.

P = 6h – 2h + 2

Effective Mathematics Teaching Practices

Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*
Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*
Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*
Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*
Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*
Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*
Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

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http://www.nctm.org/principlestoactions

NCTM | NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Teacher and Student Actions of the Mathematics Teacher Practices as outlined in NCTM's *Principles to Actions*

1. Establish Mathematics Goals to Focus Learning

What are *teachers* doing?	What are *students* doing?
Establishing clear goals that articulate the mathematics that students are learning as a result of instruction in a lesson, over a series of lessons, or throughout a unit. Identifying how the goals fit within a mathematics learning progression. Discussing and referring to the mathematical purpose and goal of a lesson during instruction to ensure that students understand how the current work contributes to their learning. Using the mathematics goals to guide lesson planning and reflection and to make in-the-moment decisions during instruction.	Engaging in discussions of the mathematical purpose and goals related to their current work in the mathematics classroom (e.g., What are we learning? Why are we learning it?) Using the learning goals to stay focused on their progress in improving their understanding of mathematics content and proficiency in using mathematical practices. Connecting their current work with the mathematics that they studied previously and seeing where the mathematics is going. Assessing and monitoring their own understanding and progress toward the mathematics learning goals.

2. Implement Tasks That Promote Reasoning and Problem Solving

What are *teachers* doing?	What are *students* doing?
Motivating students' learning of mathematics through opportunities for exploring and solving problems that build on and extend their current mathematical understanding. Selecting tasks that provide multiple entry points through the use of varied tools and representations. Posing tasks on a regular basis that require a high level of cognitive demand. Supporting students in exploring tasks without taking over student thinking. Encouraging students to use varied approaches and strategies to make sense of and solve tasks.	Persevering in exploring and reasoning through tasks. Taking responsibility for making sense of tasks by drawing on and making connections with their prior understanding and ideas. Using tools and representations as needed to support their thinking and problem solving. Accepting and expecting that their classmates will use a variety of solution approaches and that they will discuss and justify their strategies to one another.

3. Use and Connect Mathematical Representations

What are *teachers* doing?	What are *students* doing?
Selecting tasks that allow students to decide which representations to use in making sense of the problems. Allocating substantial instructional time for students to use, discuss, and make connections among representations. Introducing forms of representations that can be useful to students. Asking students to make math drawings or use other visual supports to explain and justify their reasoning. Focusing students' attention on the structure or essential features of mathematical ideas that appear, regardless of the representation. Designing ways to elicit and assess students' abilities to use representations meaningfully to solve problems.	Using multiple forms of representations to make sense of and understand mathematics. Describing and justifying their mathematical understanding and reasoning with drawings, diagrams, and other representations. Making choices about which forms of representations to use as tools for solving problems. Sketching diagrams to make sense of problem situations. Contextualizing mathematical ideas by connecting them to real-world situations. Considering the advantages or suitability of using various representations when solving problems.

4. Facilitate Meaningful Mathematics Discourse

What are *teachers* doing?	What are *students* doing?
Engaging students in purposeful sharing of mathematical ideas, reasoning, and approaches, using varied representations. Selecting and sequencing student approaches and solution strategies for whole-class analysis and discussion. Facilitating discourse among students by positioning them as authors of ideas, who explain and defend their approaches. Ensuring progress toward mathematical goals by making explicit connections to student approaches and reasoning.	Presenting and explaining ideas, reasoning, and representations to one another in pair, small-group, and whole-class discourse. Listening carefully to and critiquing the reasoning of peers, using examples to support or counterexamples to refute arguments. Seeking to understand the approaches used by peers by asking clarifying questions, trying out others' strategies, and describing the approaches used by others. Identifying how different approaches to solving a task are the same and how they are different.

5. Pose Purposeful Questions

What are *teachers* doing?	What are *students* doing?
Advancing student understanding by asking questions that build on, but do not take over or funnel, student thinking. Making certain to ask questions that go beyond gathering information to probing thinking and requiring explanation and justification. Asking intentional questions that make the mathematics more visible and accessible for student examination and discussion. Allowing sufficient wait time so that more students can formulate and offer responses.	Expecting to be asked to explain, clarify, and elaborate on their thinking. Thinking carefully about how to present their responses to questions clearly, without rushing to respond quickly. Reflecting on and justifying their reasoning, not simply providing answers. Listening to, commenting on, and questioning the contributions of their classmates.

6. Build Procedural Fluency from Conceptual Understanding

What are *teachers* doing?	What are *students* doing?
Providing students with opportunities to use their own reasoning strategies and methods for solving problems. Asking students to discuss and explain why the procedures that they are using work to solve particular problems. Connecting student-generated strategies and methods to more efficient procedures as appropriate.	Making sure that they understand and can explain the mathematical basis for the procedures that they are using. Demonstrating flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems. Determining whether specific approaches generalize to a broad class of problems.
Using visual models to support students' understanding of general methods. Providing students with opportunities for distributed practice of procedures.	Striving to use procedures appropriately and efficiently.

7. Support Productive Struggle in Learning Mathematics

What are *teachers* doing?	What are *students* doing?
Anticipating what students might struggle with during a lesson and being prepared to support them productively through the struggle. Giving students time to struggle with tasks, and asking questions that scaffold students' thinking without stepping in to do the work for them. Helping students realize that confusion and errors are a natural part of learning, by facilitating discussions on mistakes, misconceptions, and struggles. Praising students for their efforts in making sense of mathematical ideas and perseverance in reasoning through problems.	Struggling at times with mathematics tasks but knowing that breakthroughs often emerge from confusion and struggle. Asking questions that are related to the sources of their struggles and will help them make progress in understanding and solving tasks. Persevering in solving problems and realizing that is acceptable to say, "I don't know how to proceed here," but it is not acceptable to give up. Helping one another without telling their classmates what the answer is or how to solve the problem.

8. Elicit and Use Evidence of Student Thinking

What are *teachers* doing?	What are *students* doing?
Identifying what counts as evidence of student progress toward mathematics learning goals. Eliciting and gathering evidence of student understanding at strategic points during instruction. Interpreting student thinking to assess mathematical understanding, reasoning, and methods. Making in-the-moment decisions on how to respond to students with questions and prompts that probe, scaffold, and extend. Reflecting on evidence of student learning to inform the planning of next instructional steps.	Revealing their mathematical understanding, reasoning, and methods in written work and classroom discourse. Reflecting on mistakes and misconceptions to improve their mathematical understanding. Asking questions, responding to, and giving suggestions to support the learning of their classmates. Assessing and monitoring their own progress toward mathematics learning goals and identifying areas in which they need to improve.

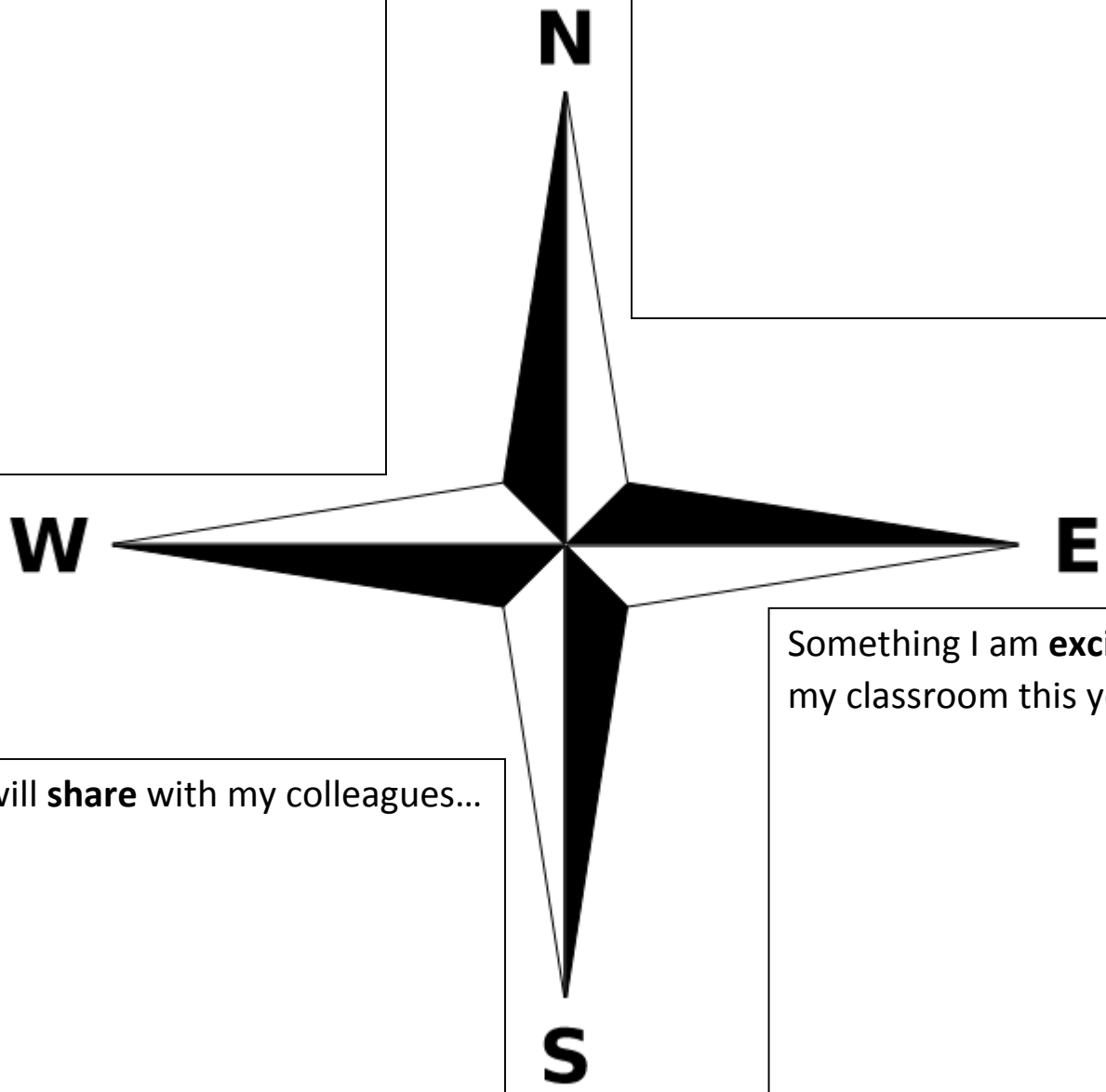
Note-Capturing Tool

Mathematics Teaching Practices	Evidence of the Practice
Establish mathematics goals to focus learning. *Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.*	
Implement tasks that promote reasoning and problem solving. *Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.*	
Use and connect mathematical representations. *Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.*	
Facilitate meaningful mathematical discourse. *Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.*	

Pose purposeful questions. *Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.*	
Build procedural fluency from conceptual understanding. *Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.*	
Support productive struggle in learning mathematics. *Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.*	
Elicit and use evidence of student thinking. *Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.*	

Something I am still **wondering** about…

Something **new** and/or **noteworthy** that I don't want to forget…



Something I am **excited** about doing in my classroom this year…

Something that I will **share** with my colleagues…

NYC Department of Education
Carmen Fariña, Chancellor

Mathematics Supplemental Resources

1. Full Progressions Documents for the Common Core Math Standards
   (http://ime.math.arizona.edu/progressions)

2. Illustrative Mathematics: Common Core Math Tasks
   (https://www.illustrativemathematics.org)

3. Inside Mathematics: Common Core Task & Video Collection
   (www.insidemathematics.org)

4. Mathematics Assessment Project: Tasks, Assessments & Lesson Collection
   (http://map.mathshell.org)

5. Achieve the Core: Resources for Common Core Implementation
   (http://achievethecore.org)

6. National Council of Teachers of Mathematics: Principals to Actions Toolkit
   (http://www.nctm.org/PtAToolkit/)

7. Strategic Education Research Partnership's Math Resources
   (http://math.serpmedia.org)

8. NYSED EngageNY: Math Curriculum & Scope and Sequence (Grades K-12)
   (https://www.engageny.org/common-core-curriculum)

9. Teaching Channel: Video Collection by Subject, Grade & Topic
   (https://www.teachingchannel.org)