COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

HIGH SCHOOL ALGEBRA: AUSSIE FIR TREE

UNIT OVERVIEW

The Aussie Fir Tree task is a culminating task for a 2-3 week unit on algebra that uses the investigation of growing patterns as a vehicle to teach students to visualize, identify and describe real world mathematical relationships. Students who demonstrate mastery of the unit are able to solve the Aussie Fir Tree task in one class period.

TASK DETAILS

Task Name: Aussie Fir Tree

Grade: High School

Subject: Algebra

Task Description: This task asks students to recognize geometric patterns, visualize and extend the pattern, generate a non-linear sequence, develop and algebraic generalization that models the growth of a quadratic function and verify the inverse relationship of the quadratic relationship.

Standards Assessed:
F.BF.1 Write a function that describes a relationship between two quantities.
F.BF.1a Determine an explicit expression, a recursive process, or steps for calculation from a context.
F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
A.CED.1 Create equations in one variable and use them to solve problems. Include equations arising from linear and quadratic functions.
A.REI.4 Solve quadratic equations in one variable.

Standards for Mathematical Practice:
MP.1 Make sense of problems and persevere in solving them.
MP.3 Construct viable arguments and critique the reasoning of others.
MP.4 Model with mathematics.
MP.6 Attend to precision.
MP.7 Look for and make use of structure.

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HIGH SCHOOL ALGEBRA: AUSSIE FIR TREE PERFORMANCE TASK

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Dennis M. Walcott, Chancellor

The Aussie Fir Tree

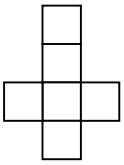
It grows down under

Consider the following function that generates the geometric pattern of a reverse growing fir tree.

Stage 1

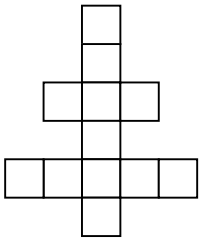
2 unit squares

Stage 2



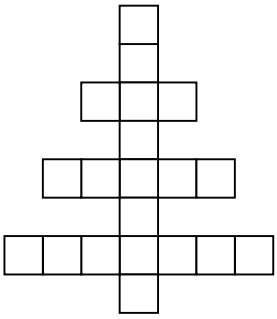
6 unit squares

Stage 3



12 unit squares

Stage 4



1. Draw and describe **Stage 5** of the pattern in terms of its shape and number of unit squares needed to construct the fir tree.

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The Aussie Fir Tree

It grows down under

2. Describe how the pattern is growing?

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

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HIGH SCHOOL ALGEBRA: AUSSIE FIR TREE RUBRIC

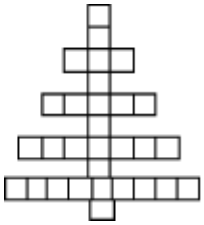
The rubric section contains a scoring guide and performance level descriptions for the Aussie Fir Tree task.

Scoring Guide: The scoring guide is designed specifically to each small performance task. The points highlight each specific piece of student thinking and explanation required of the task and help teachers see common misconceptions (which errors or incorrect explanations) keep happening across several papers. The scoring guide can then be used to refer back to the performance level descriptions.

Performance Level Descriptions: Performance level descriptions help teachers think about the overall qualities of work for each task by providing information about the expected level of performance for students. Performance level descriptions provide score ranges for each level, which are assessed using the scoring guide.

High School Algebra: Aussie Fir Tree
Rubric

Aussie Fir Tree Scoring Guide

The Aussie Fir Tree		Rubric
The elements of performance required by this task are: • Recognizes geometric patterns. · Visualizes, extends and describes patterns. · Determines a solution to a polynomial relationship. · Develops an algebraic equation that models the growth of a quadratic function. · Verifies the inverse relationship of the polynomial equation.	Points	Section Points
1. Draws stage 5 with 30 unit squares. Describes stage 5 such as: The trunk is ten unit squares tall and there are four sets of branches 3, 5, 7 and 9 units. 	1 1	2
2. Describe the growing pattern such as: It grows by consecutive even numbers from stage to stage. The pattern grows between each stage by 2, 4, 6, 8,…	2	2
3. **110** unit squares. Show work such as: Stage 5 is 30 or 5 • 6, Stage 6 is 42 or 6 • 7, Stage 7 is 56, or 7•8 so, Stage 10 is 10 • 11 = 110 unit squares.	1 1	2
4. **Unit squares = n(n+1)** *Partial Credit* Provide a correct expression	2 (1)	2
5. Provides an explanation such as: No two consecutive integers when multiplied together equals 274. The consecutive integers 16 • 17 = 272. The next smallest product would be 17 • 18 = 306, which is too large.	2	2
Total Points		**10**

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High School Algebra: Aussie Fir Tree
Rubric

Performance Level Descriptions and Cut Scores

Performance is reported at four levels: 1 through 4, with 4 as the highest.

Level 1: Demonstrates Minimal Success (0-3 points)

The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The student struggles to recognize patterns or the structure of the problem situation.

Level 2: Performance below Standard (4-5 points)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem, it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Level 3: Performance at Standard (6-7 points)

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. The student often uses abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Level 4: Achieves Standards at a High Level (8-10 points)

The student's response meets the demands of nearly the entire task, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in construction arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. Students look closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

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COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

HIGH SCHOOL ALGEBRA: AUSSIE FIR TREE ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points and implications for instruction for each performance level. The student work shows examples of student understandings and misunderstandings of the task, which can be used with the implications for instruction to understand how to move students to the next performance level. 1

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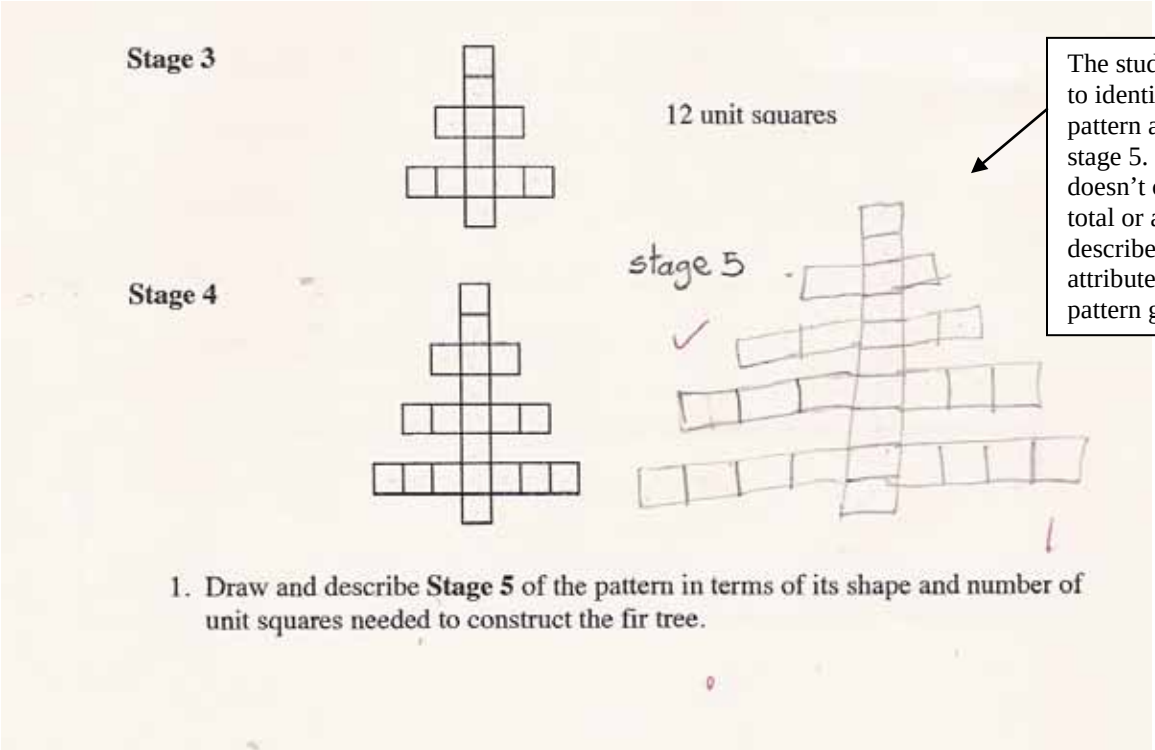
High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 4: Achieves Standards at a High Level (Score Range 8 – 10)

The student's response meets the demands of nearly the entire task, with few errors. With some more time for checking and revision, excellent solutions would seem likely. The student response shows understanding and use of stated assumptions, definitions and previously established results in construction arguments. The student is able to make conjectures and build a logical progression of statements to explore the truth of their conjecture. The student response routinely interprets their mathematical results in the context of the situation and reflects on whether the results make sense. The communication is precise, using definitions clearly. The students looks closely to discern a pattern or structure. The body of work looks at the overall situation of the problem and process, while attending to the details.

Student A (9 points)



The student is able to identify the visual pattern and draw stage 5. The student doesn't count the total or attempt to describe the attributes of how the pattern grows.

High School Algebra: Aussie Fir Tree
Annotated Student Work

2. Describe how the pattern is growing?

from the stage 1 to stage 2 "4" is added, from stage 2 to stage 3 "6" is added, from stage 3 to stage 4 "8" is added. ②

In part 2 the student notices that the pattern grows by consecutive even numbers.

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work.

Stage	unit squares
5	30
6	42
7	56
8	72
9	90
10	110

we need 110 unit squares to build stage 10. because from stage 1 to 2 you added a 4 from stage 2 to 3 you added a 6 and so on. you keep adding a 2 to the number you added.

The student uses this pattern of increasing consecutive even numbers to find the total number of squares to build stage 10.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

$\boxed{y = (n+1) \cdot n}$ ①

The student uses correct algebraic notation to write a closed equation in part 4. (CCSS Creating Equations)

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

Performance Task The Aussie Fir Tree P 2

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His statement is false because none of the stage number can have 274 unit. The stage number 16 has 272 unit but that less than 274 unit. And stage number 17 has 288 unit and that's more than 274 unit.

In part 5 the student uses the equation and substitution to show why 274 is not a possible solution. A better argument would have also included the pattern above as well the pattern below to disprove the statement. (MP 3)

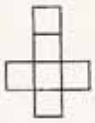
High School Algebra: Aussie Fir Tree
Annotated Student Work

Student B (8 points)

Stage 1 x2

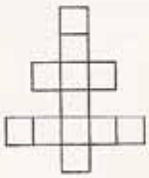
2 unit squares

Stage 2 x3



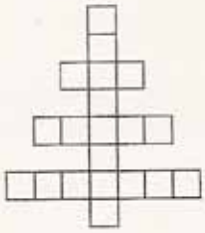
6 unit squares

Stage 3 x4



12 unit squares

Stage 4 x5



20 units

1. Draw and describe **Stage 5** of the pattern in terms of its shape and number of unit squares needed to construct the fir tree.

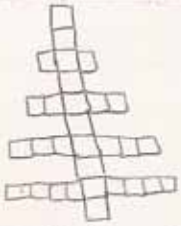
Stage 5 has one more branch with an extra square on each side. 0 1

Stage 5 = 5x6 = 30 units. ✓ 1

The student uses the information given on page one to look at patterns and to experiment with numerical expressions to get the given totals. The student then uses the pattern to generate the total squares without counting. The student is able to identify the visual pattern.

High School Algebra: Aussie Fir Tree
Annotated Student Work

Stage 5
5x6



30 units

2. Describe how the pattern is growing?

Each branch has one more unit on each side and one extra on the bottom. 2
The stage # multiplied by stage # + 1 gives the amount of unit squares needed

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work.

$n(n+1)$

Stage 10 → $10 \times 11 = 110$ units 1 1

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

$n(n+1) = a$ 2

a = amount of unit squares needed.

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

Because no 2 consecutive #s multiply to equal 274. 2 0 1

Performance Task The Aussie Fir Tree P 2

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In part 2 the student jumps right to the generalization.

The student uses the generalization (from part 2) to calculate the total for stage 10.

The student uses variables and correct notation to write a closed equation in part 4. (CCSS Creating Equations) Notice that the student defines the variable not given in the stem of the prompt.

The student could give more detail in part 5, making a convincing argument. (MP 3)

Level 4 Implications for Instruction

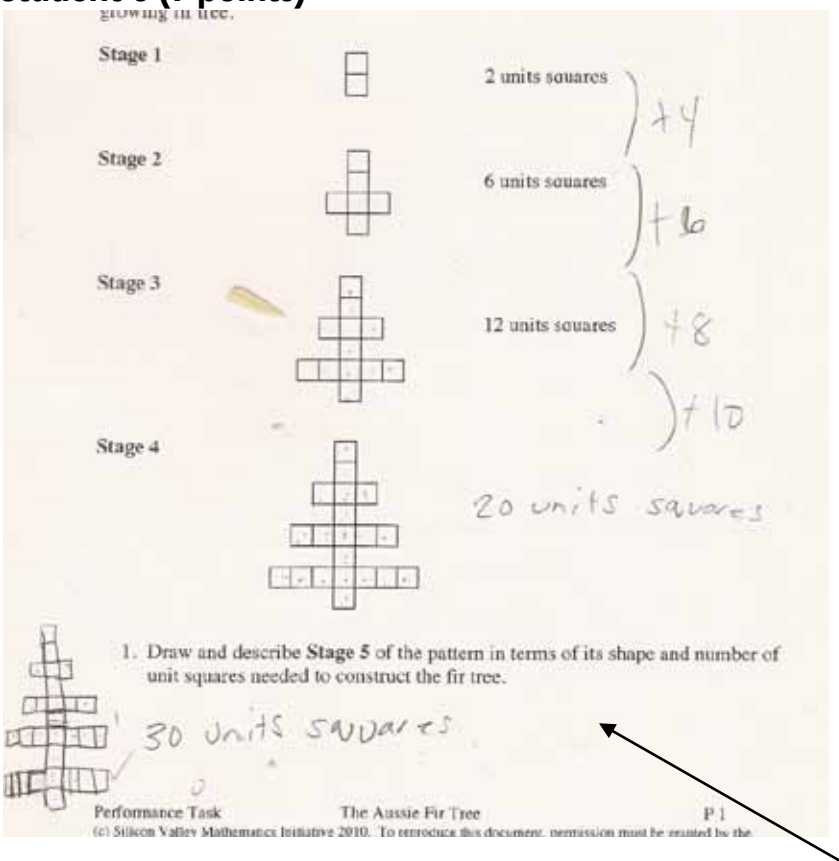
Students need more opportunities to develop justification and make convincing arguments. They should have frequent opportunities to question, to critique, and to improve the arguments of others. *"Does this convince you? Why or Why not? What would make it more convincing? Why do you disagree? How could you convince the person to change their mind?"*
Students at this level should be encouraged to give algebraic as well as numeric ideas to make their justifications.

High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 3: Performance at Standard (Score Range 6 – 7)

For most of the task, the student's response shows the main elements of performance that the tasks demand and is organized as a coherent attack on the core of the problem. There are errors or omissions, some of which may be important, but of a kind that the student could well fix, with more time for checking and revision and some limited help. The student explains the problem and identifies constraints. The student makes sense of quantities and their relationships in the problem situations. They often use abstractions to represent a problem symbolically or with other mathematical representations. The student response may use assumptions, definitions, and previously established results in constructing arguments. They may make conjectures and build a logical progression of statements to explore the truth of their conjectures. The student might discern patterns or structures and make connections between representations.

Student C (7 points)



The student is able to identify the visual pattern and count the squares in part 1, but does not attempt to describe how the attributes of the shape grow.

High School Algebra: Aussie Fir Tree
Annotated Student Work

In part 2 the student notes clearly the pattern of increasing consecutive even numbers and can use this pattern to make a convincing argument in part 5.

The Aussie Fir Tree
It grows down under

2. Describe how the pattern is growing?

2 +4 → 6 +6 → 12 +8 → 20

The Pattern is quodratatic, and it is increasing by 2 in the Second diffrency.

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work.

2 +4 6 +6 12 +8 20 +10 30 +12 42 +14 56 +16 72 +18 90 +20 110

Stage 10 = 110

The student is also able to use the pattern of increasing consecutive even numbers to find the total for stage 10.

However the student can't move past the recursive pattern to find an equation in part 4. (CCSS Equations and Inequalities) The student seems to attempt to quantify an algebraic pattern for stage 3, which does not correlate to work in other parts of the paper. The student doesn't seem to connect the idea of equation as a tool for generalizing to all cases.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

Stage # = 3 n+3 = X

I put stage 3 for an example and added it by n so the answer will be X until I find what n equals.

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

110 +22 132 +24 156 +26 182 +28 210 +30 240 +32 272

You can't make one because it would be 2 units short.

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High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 3 Implications for Instruction

Students at this level still struggle with giving good verbal descriptions of what they see. The biggest difficulties were in finding generalizable rules rather than recursive rules. One helpful way to do this is to ask students break down the pattern into simpler parts. "*What do you see when you look at the pattern? How can you decompose the shape into parts?. . . . How does the tree trunk grow? How could we use algebra to describe how to find the tree trunk for any pattern number?"* Breaking down the problem into simpler steps helps students manage the thinking in smaller chunks.

Students need opportunities to compare and contrast strategies and to evaluate their usefulness. So during class discussions it is important not to stop after several students have shared how they solved the problems. Teachers need to ask students to think about and reflect on the strategies. "*Which one is easier to use? Why? Which one would be most useful for finding the 1000th term? Why? What is the same about both formulas or strategies? What is different? How can we be sure that they will always give the same solution?"*

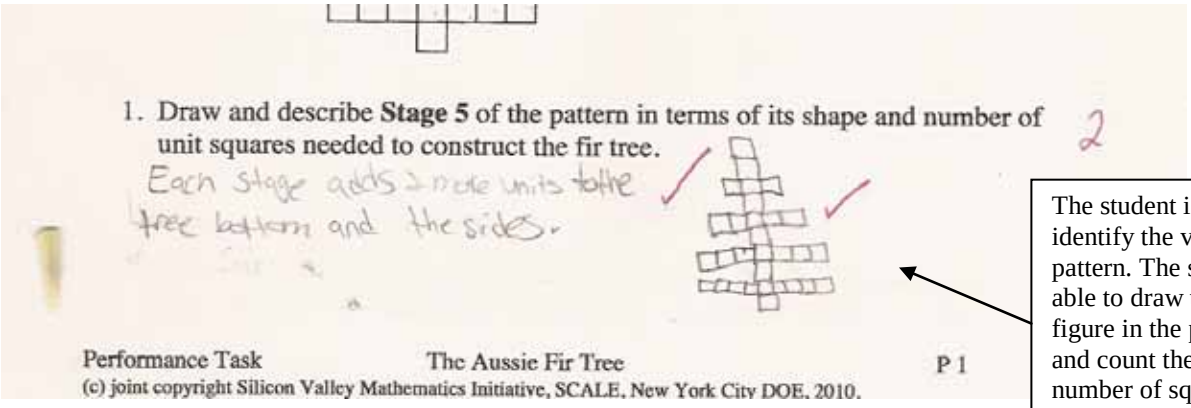
Students need more opportunities to develop justification and make convincing arguments. They should have frequent opportunities to question, to critique, and to improve the arguments of others. "*Does this convince you? Why or Why not? What would make it more convincing? Why do you disagree? How could you convince the person to change their mind?"*

High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 2: Performance below Standard (Score Range 4-5)

The student's response shows some of the elements of performance that the tasks demand and some signs of a coherent attack on the core of some of the problems. However, the shortcomings are substantial, and the evidence suggests that the student would not be able to produce high-quality solutions without significant further instruction. The student might ignore or fail to address some of the constraints. The student may occasionally make sense of quantities in relationships in the problem, but their use of quantity is limited or not fully developed. The student response may not state assumptions, definitions, and previously established results. While the student makes an attack on the problem it is incomplete. The student may recognize some patterns or structures, but has trouble generalizing or using them to solve the problem.

Student D (4 points)



1. Draw and describe **Stage 5** of the pattern in terms of its shape and number of unit squares needed to construct the fir tree.

2

Each stage adds 2 more units to the tree bottom and the sides.

Performance Task The Aussie Fir Tree P 1

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The student is able to identify the visual pattern. The student is able to draw the next figure in the pattern and count the total number of squares in the drawing. The student notices that the middle row increases by 2 each time and that each new branch adds another square on each side.

High School Algebra: Aussie Fir Tree
Annotated Student Work

The student isn't quite able to come up with the idea of adding consecutive even numbers each time, but gives a minimal description tied to the structure of the visual pattern.

2. Describe how the pattern is growing? ✓ 2

The tree grows with 2 units added to the bottom at every stage. However many units were previously at the side there's also 2 more units added to that number to put at the next stage.

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work. x 0

Stage 9 - 78 units

To build a stage 10 Aussie Fir Tree you need 98 unit squares.

78
+20
98

Stage 10 - 98

The student also shows this understanding of consecutive even numbers by choosing to add 20 to stage 9 to get stage 10. Unfortunately the student has not shown enough work, but appears to have made a calculation error somewhere between stage 5 and stage 9.

The student does not attempt to write an equation. In part 3 he uses a recursive idea of adding 2, which matches to either increasing the side of the branch, but does not include the idea that new branches are added, or thinks about the vertical branch increasing by 2. (CCSS Seeing Structure in Expressions) The student needs substantial instruction to think about writing an equation for the entire situation.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree. x 0

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false. x 0

The recursive thinking is continued in part 5. (CCSS Creating Equations)

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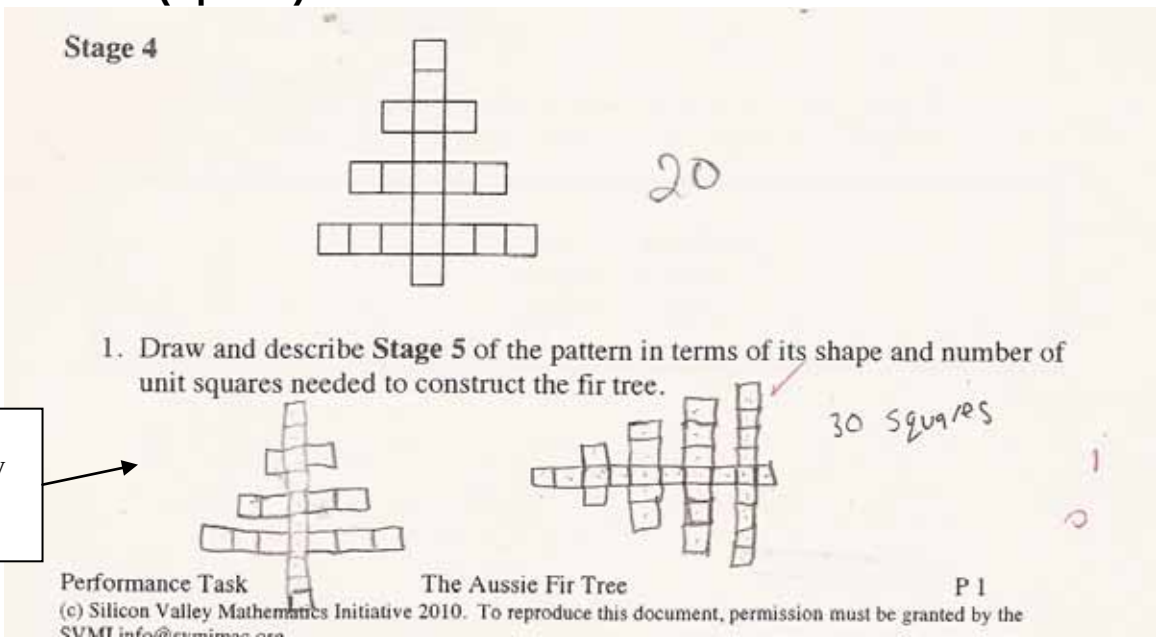
He is wrong because he needs to know what stage and the number of units from the previous stage to know how much unit squares makes up one tree/stage.

4

High School Algebra: Aussie Fir Tree
Annotated Student Work

Student E (5 points)



Stage 4

20

1. Draw and describe **Stage 5** of the pattern in terms of its shape and number of unit squares needed to construct the fir tree.

30 squares

Performance Task The Aussie Fir Tree P 1

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The student is able to identify the visual pattern.

High School Algebra: Aussie Fir Tree
Annotated Student Work

The student is much more descriptive in the size of the sides, stage number minus one. This quantification sets a foundation for writing the equation.

2. Describe how the pattern is growing?

It is adding two more squares down but on the top of the two it is adding the amount of square equal to the stage number minus one on each side.

2

3. How many unit squares are needed to build a **Stage 10** Aussie Fir Tree? Show your work. It would need 110 squares.

x	y
1	2
2	6
3	12
4	20
5	30

+4, +6, +8, +10

x	y
5	30
6	42
7	56
8	72
9	90
10	110

+12, +14, +16, +18, +20

1

1

The student chooses to use a table to extend the pattern of adding consecutive even numbers to total squares for stage 10.

4. Given any stage number **n**, determine a closed form equation to determine the amount of unit squares needed to build the tree.

x = stage number
y = total # of squares

$x^2 + (x \cdot = y$

0 2

The student can then use the information from the table to write an equation for the pattern. (MP5)

11
× 11
11
11
121

5. Your mate tells you that exactly 274 unit squares will make an Aussie Fir Tree. He is wrong. Explain to him why his statement is false.

His statement is wrong because if you use the equation above then it wouldn't be able to do the equation backwards. the closest to 274 units would be 280. That would mean it wouldn't make a Aussie fir tree. This is proven is the table on the left.

x	y
10	110
11	132
12	164
13	190
14	218
15	248
16	280

+22, +24, +26, +28, +30, +32

x 0

5

The student uses addition to prove why the statement is incorrect in part 5, which could have led to the convincing argument without the small arithmetic error. (MP 3) The student didn't check the rule using the equation generated in part 4. (MP 1)

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High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 2 Implications for Instruction

Many students at this level are using recursive rules to extend the pattern. While this is helpful and easy to use for a small number of cases, it is cumbersome and prone to errors when trying to extend the pattern for larger numbers. Students need to see that it is more helpful to find more generalizable rules to solve problems for all cases.

One helpful way to do this is to ask students break down the pattern into simpler parts. "*What do you see when you look at the pattern? How can you decompose the shape into parts? How does the tree trunk grow? How could we use algebra to describe how to find the tree trunk for any pattern number?"* Breaking down the problem into simpler steps helps students manage the thinking in smaller chunks. But students in this stage need to learn questions to help them progress in their thinking, to develop strategies for finding a generalizable rule.

Students need to be encouraged to give more detail about what they see. So in class the teacher might ask, "*That's interesting, can you tell me a bit more?* **Or** *where do you see the n in the diagram or the (n+1)?"* The more detailed their descriptions usually the easier it is to quantify the ideas symbolically. Students should also be more descriptive in thinking about classes of numbers. In elementary school it is good to notice that numbers are odd or even, but by this grade level students should start to classify numbers as consecutive or consecutive odd numbers, multiples of . . ., powers of . . . , triangular numbers, etc. The types of patterns that students think about should be expanded.

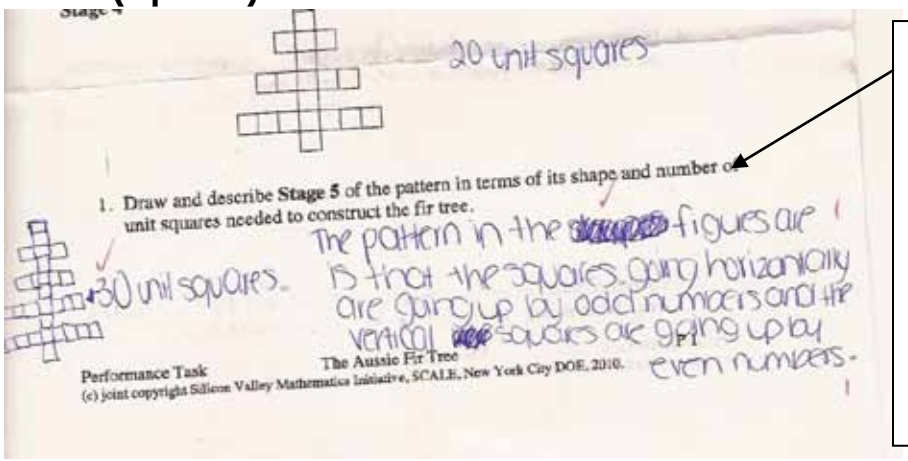
Students need to have experiences thinking about types of linear patterns; those that are proportional and those that are not proportional. Take the work of student 692. The strategy of doubling from case 5 to case 10 works for proportional patterns, but not for patterns with a constant. Students can benefit by looking at two cases at the same time and comparing which one will work by doubling and which one won't. Drawing graphs of the situations can help to clarify this idea.

High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 1: Demonstrates Minimal Success (Score Range 0 –3)

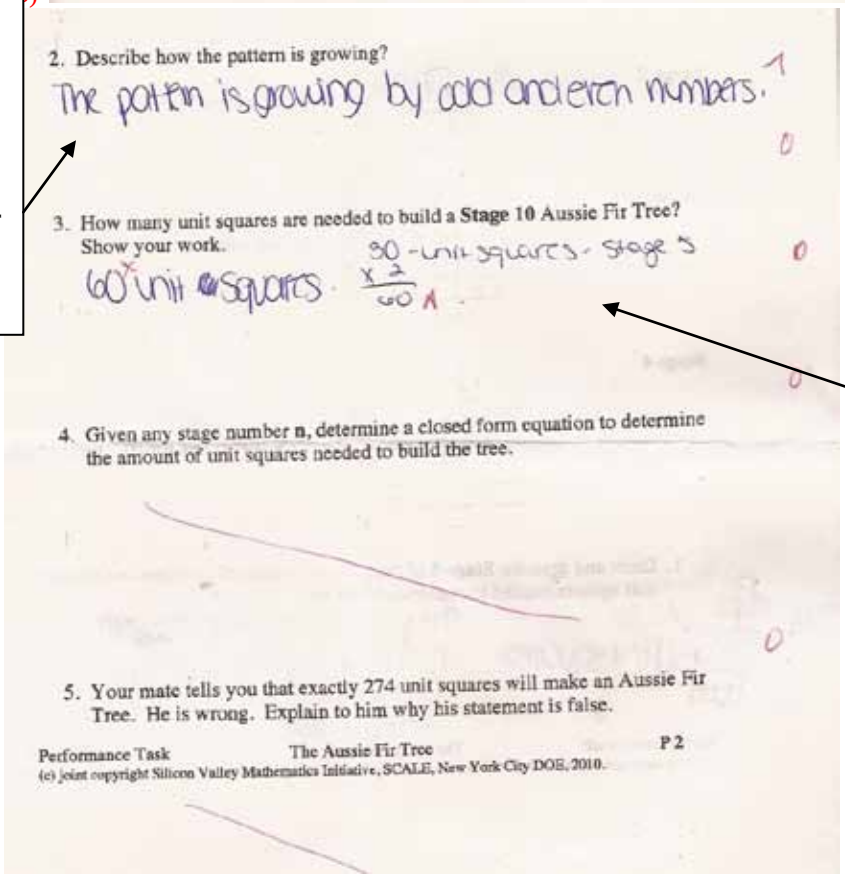
The student's response shows few of the elements of performance that the tasks demand. The work shows a minimal attempt on the problem and struggles to make a coherent attack on the problem. Communication is limited and shows minimal reasoning. The student's response rarely uses definitions in their explanations. The student struggles to recognize patterns or the structure of the problem situation.

Student F (2 points)



The student is able to identify the visual pattern. The student is able to draw and count the squares in the next pattern of the sequence. The student notices that each new branch is going up by odd numbers, but doesn't quantify how much to increase each time.

The student notices that the vertical squares go up by an even number, rather than increase by 2's. By grade 9, students should be digging deeper than odd or even for patterns. (MP 1)



The student uses proportional reasoning to increase the pattern from stage 5 to stage 10. The lack of quantitative reasoning at earlier parts of the task doesn't give the student enough detail to see that the pattern is not proportional or even linear. (MP 7 CCSS Seeing Structure in Expressions) The student does not have the perseverance to attempt further parts of the task.

High School Algebra: Aussie Fir Tree
Annotated Student Work

Level 1 Implications for Instruction

Students need help learning to describe accurately and precisely what they see as the visual pattern grows. These descriptions become tools that can be used to describe the pattern algebraically.

Students need ways of organizing their thinking, such as making tables to see how the pattern grows numerically. The work done by student 209 can help to lead to a full solution algebraically, but students need discussions about the purpose of the tools. Students at this level don't know or understand the purpose of the tools, so even when they make a table that aren't sure of what they learn from it that can help them make the generalization. A useful teaching devise is self talk. The teacher starts to solve a problem or a student describing a solution goes part way, then the teacher stops and asks, "What do you think comes next?" Students need to start anticipating how the solution progresses. In this case, students might notice that each stage increases by a consecutive even number.

Students need to talk about cases. When making a conjecture, they need to understand that just one example is not sufficient. They need to test their ideas against several cases to see if it at least holds true for all the examples that they have available. At later stages of their development, they will learn that there are never enough examples to prove a case and that the proof lies in the physical geometry of the pattern. However, many students at this level made a generalization, when they had other examples on their paper that disproved their assertions. Students need frequent opportunities to work with patterns, organize the information for them, and make and test conjectures.