COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 11 MATH: FERRIS WHEEL TASK

UNIT OVERVIEW

Adhering to the traditional pathway guidelines for Common Core Algebra II, this unit covers the trigonometric Common Core standards associated with the Algebra II course (TF.B.5, TF.A.1, TF.A.2, TF.C.8). Students will begin the unit by studying the unit circle (TF.A.1 and TF.A.2). In Geometry, students already learned to find the degree measure from a radian measure and vice versa (C.B.2). In this unit students learn the definition of what a radian is and use that definition to find the arc length, radius, or radian. Students also learn about how to use the unit circle with trigonometric functions. Once students understand the unit circle and its relationship with trigonometric functions, they will then learn about the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle (TF.C.8). The final topic will be modeling using trigonometric functions (TF.B.5).

TASK DETAILS

Task Name: Ferris Wheel Task

Grade: 11

Subject: Math

Depth of Knowledge: 3

Unit/Sub-Unit Description: In Geometry, students began studying trigonometric ratios (G.SRT.C.6-8). This unit builds off of those standards. Students will be introduced to the unit circle. Students will study how to use the unit circle (TF.A.1, 2) to find trigonometric function values for particular radian and degree measures. This will give teachers an opportunity to assess students' prior knowledge of the trigonometric ratios. Tasks for this sub-unit will focus primarily on students having fluency in using the unit circle. The following sub-unit will have students manipulate trigonometric ratios using the Pythagorean identity (TF.C.8). Students will be introduced to frequency and will start graphing trigonometric functions. The tasks in this sub-unit will involve students explaining their reasoning and teachers will need to model for their students how to explain their steps. Students will learn about the amplitude and midline for trigonometric graphs, and will learn how to model periodic phenomena using trigonometric functions (TF.B.5). Tasks for this sub-unit will have little prompts and require more of the math practices than the other sub-units.

Task Description: Students are presented with information about the dimensions of an actual Ferris wheel. They must come up with a model for the height of a cart on the Ferris wheel at particular times. While most Ferris wheel problems give students the initial height, in this task students must make assumptions about the initial height with

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the information provided. This might mean that some of the answers for the midline will vary; however, the frequency and amplitude of the function should be the same as the rubric. The task requires students to explain their reasoning for their steps so that teachers have more evidence of students' content knowledge and math practices. Students are also asked to answer several questions that address the other Algebra II trigonometric function standards.

Standards Assessed:

F-TF.B.5: Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

F-TF.A.1: Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

F-TF.A.2: Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

F-TF.C.8: Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos(θ), or tan(θ), given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.

Standards for Mathematical Practice:

Assessed by the task:

MP.1: Preserve through problem solving.
MP.2: Reason abstractly and quantitatively.
MP.4: Model with mathematics.*

**Not on the rubric, but students have the possibility to use this practice when completing the task.*

Assessed in the unit but not the task:

MP.3: Construct viable arguments and critique the reasoning of others.

TABLE OF CONTENTS

The task and instructional supports in the following pages are designed to help educators understand and implement Common Core–aligned tasks that are embedded in a unit of instruction. We have learned through our pilot work that focusing instruction on units anchored in rigorous Common Core–aligned assessments drives significant shifts in curriculum and pedagogy.

Acknowledgements: This bundle was developed by Noam Pillischer, Jessica Sherman, and Sandy Ting at Urban Assembly Institute of Math and Science for Young Women (13K527), in collaboration with the Common Core Fellows.

GRADE 11 MATH: FERRIS WHEEL

PERFORMANCE TASK

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Cumulating Task

Ferris Wheel Task

Renny is a Ferris Wheel fanatic. She saved and bought a ticket to Japan to go on the Palallete Town Ferris Wheel (pictured below).

She went online and got the following information:

Height: 115 meters
Diameter: 100 meters
Rotation speed: 16 minutes/revolution
Location: Odaiba, Tokyo, Japan
Opening: 1999

Renny wants to tweet how high she is above the ground every 5 minutes. She decides that she will have to figure out how high she will be before hand. How high will Renny be at five minutes, ten minutes and 15 minutes? Provide a written justification of your solution, supported by mathematical evidence.

Standards: F.TF.B.5
Math Practices: 1, 2, 4

1) Draw and label a diagram of the ferris wheel.

2) Calculate the angle of rotation *per minute*.

3) Create a trigonometric function to model Renny's height over time.

4) Solve for her height at 5, 10, and 15 minutes.

5) Explain your strategy and state your solution.

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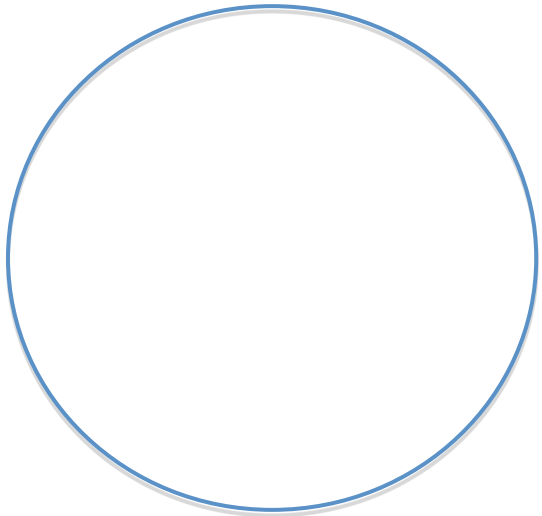
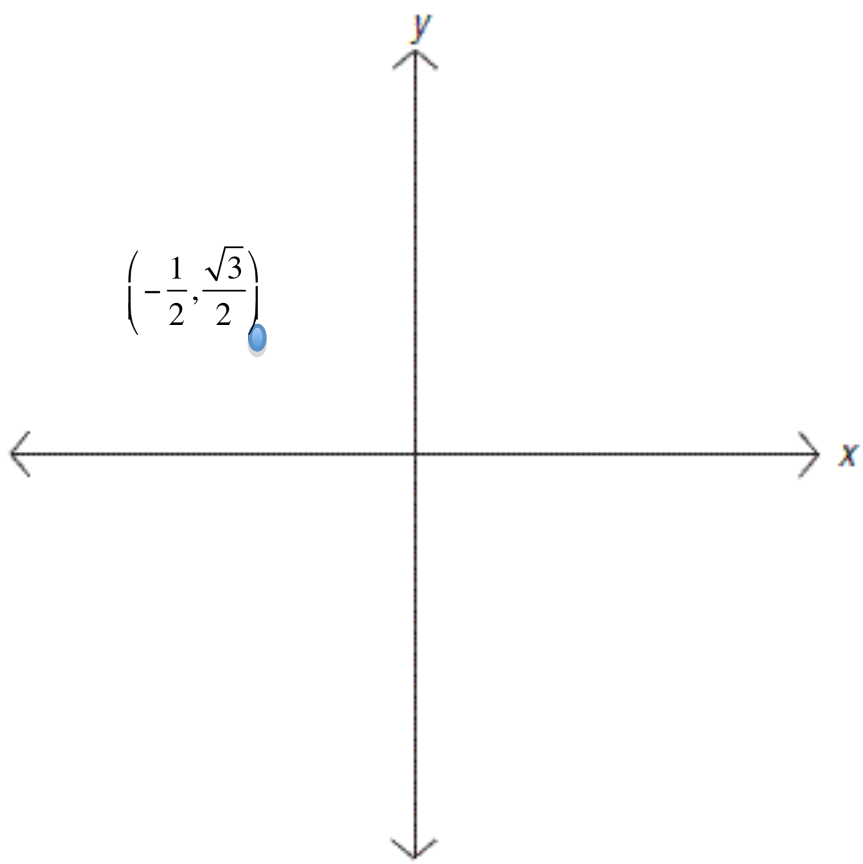
What is a Radian?

Using the circle below draw and explain what a radian is.



How long is the arc subtended by an angle of $\frac{4\pi}{3}$ radians on a circle of radius 15cm?

Find both the radian and degree measure of the angle between the point on the coordinate plane below and x-axis without the use of a calculator. Show and explain your work.



Pythagorean Identity

Explain why $\sin\Theta^2+\cos\Theta^2 = 1$

Using the Pythagorean identity, find the value of $\sin\Theta$ and $\tan\Theta$ if $\cos\Theta = -\frac{1}{2}$

GRADE 11 MATH: FERRIS WHEEL

SCORING GUIDE AND RUBRIC

Modeling Rubric

Indicators	Mastery with Distinction	Mastery	Satisfactory	Needs Improvement	Did Not Attempt
<u>HSF-TF.B.5</u> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	1. Student achieves mastery. 2. Student identifies the amplitude, frequency, and midline. Student also relates the three descriptions to the context of the task.	1. Student: a. Models Ferris wheel motion with the following function: 65-50cos(22.5t). b. Shows steps how he/she got the solution, with no errors. c. States the height and time for the three times (5,84.13), (10,100.35), and (15, 18.81).	1. Student recognizes that the Ferris wheel's height and time can be represented as a cosine function. 2. Student is able to derive two out of the three function characteristics (amplitude, frequency, and midline).	1. From the work it is evident that the student has major content standard misconceptions and needs immediate intervention.	
<u>MP.1</u> Make sense of problems and persevere in solving them. **Evidence:** ☐ **Math Work** ☐ **Writing**	1. Student achieves mastery. 2. Student checks his or her work and shows that the solution makes sense with supporting evidence.	1. Student is able to identify an entry point for the solution of the task. 2. Student identifies constraints, relationships, and goals. 3. Student makes a conjecture about the form and meaning of a possible solution and plans a solution pathway that leads to the solution of the task.	1. Student is able to identify entry point for the solution of the task. 2. Student identifies some constraints, relationships, and goals related to the task. 3. Student attempts to make a connection between the task and the content standard.	1. Student is only able to identify a starting point for the solution of the task.	
<u>MP.2</u> Reason abstractly and quantitatively. **Evidence:** ☐ **Math Work** ☐ **Writing**	1. Student achieves mastery. 2. Student recognizes that some assumptions were made, specifically the starting position of the cart. Student explains that there is a margin of error with the solution provided.	1. Through student's work it is shown that he/she is able to decontextualize the task and represent it symbolically and manipulate the representing symbols. Later in the problem-solving process, the student is able to contextualize and connect his/her math work with the context of the task. 2. Student considers the units (minutes, revolutions) involved in the task and attends to their meaning.	1. Students demonstrate through either his/her writing or math work the ability to decontextualize the task as well as later contextualizing the math work. However, student does not do this on a consistent basis, and there are some errors in the work or in the writing.	1. There is no evidence of student decontextualizing the task. 2. Work suggests that the student was trying to connect to the topic of the unit to the task but didn't know how or why. Student was relying more on test-taking savvy than math knowledge.	

Radian/Unit Circle Rubric

Indicators	Mastery with Distinction	Mastery	Satisfactory	Needs Improvement	Did Not Attempt
CCSS.Math.Content.HSF-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	1. Student achieves mastery. *2.* **In student's written work:** Student explains his/her work with precise language.	1. **In student's math work:** Student recognizes that the point is on the unit circle and therefore the cosine and sine equal *x*- and *y*-axis, respectfully. Student uses the special triangle relationships and the $\pi/2$ movement to find the angle between the *x*-axis and the point. 2. **In student's written work:** Student's explanation lacks some key terms, but the math work shows a strong understanding of the standard.	3. **In student's math work:** Student recognizes that the point is on the unit circle and therefore the cosine and sine equal *x* and *y*-axis, respectfully. Student uses a calculator instead of the special triangle relationships and the $\pi/2$ movement to find the angle of between the *x*-axis and the point.	From the work it is evident that the student has major content standard misconceptions and needs immediate intervention.	
CCSS.Math.Content.HSF-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	1. Student achieves mastery. 2. Student uses a variable for the radius and arc length in the picture used to describe a radian, showing the student understands that one radian is when the length of the radius and arc length are the same for any length.	1. Student is able to give a description of what a radian measure is, along the lines of: "A radian is an angle measure for the angle at the center of a circle where the length of the arc made by the two radii is equal to the length of the radius." **AND** Student draws a sector and labels the length of the radius and arc length the same number. 2. Student is able to find the arc length of the sector, 20π, and shows proper work.	1. Student is able to give a description of what a radian measure is, along the lines of: "A radian is an angle measure for the angle at the center of a circle where the length of the arc made by the two radii is equal to the length of the radius." **OR** Student draws a sector and labels the length of the radius and arc length the same number. 2. Student is able to set up the equation to find the arc length of the sector but makes one calculation error and does not get 20π.	From the work it is evident that the student has major content standard misconceptions and needs immediate intervention.	

<u>CCSS.Math.Content.HSF-TF.C.8</u> Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.	1. Student achieves mastery. 2. Teacher deems student's explanation of how to find the Pythagorean identity to be above grade level.	1. Student's explanation of the Pythagorean identity uses precise language, is coherent, and hits all the major understandings that are needed to come up with the Pythagorean identity. 2. Student uses the Pythagorean identity for finding both tangent and sine.	1. Student's explanation of the Pythagorean identity is missing one key understanding. 2. Student uses the Pythagorean identity only once when finding sine and tangent of the angle theta.	1. Student's explanation of the Pythagorean identity is unclear or incorrect, missing all key understandings. 2. Student does not use the Pythagorean identity.	
<u>MP.6</u> Attend to precision **Evidence:** ☐ **Math Work** ☐ **Writing**	1. Student achieves mastery. 2. Student's written response is deemed by the teacher at a higher level than what is expected for the grade.	1. Student's calculations are accurate and are found efficiently. 2. Student's units and representation of the answers show a deep understanding of the topic he/she is working with. For example, leaving an answer in terms of pi instead of approximating when working with radians. 3. Student's written explanation is precise, meaning math terms are used properly and in a coherent fashion so that the reader understands the steps he/she took in finding the solution.	1. Student's calculations are accurate and are found efficiently.	1. Student makes too many mistakes when calculating, showing little attention to precision in his/her work.	

GRADE 11 MATH: FERRIS WHEEL

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points and suggested next steps for students. The student work shows examples of student understandings and misunderstandings of the task.

Cumulating Task

Ferris Wheel Task

Renny is a Ferris wheel fanatic. She saved and bought a ticket to Japan to go on the Palette Town Ferris Wheel (pictured below).

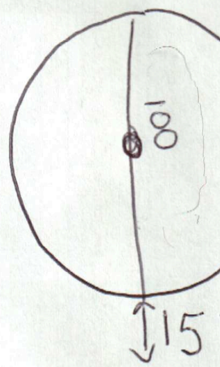
She went online and got the following information:

Height: 115 meters
Diameter: 100 meters
Rotation speed: 16 minutes/revolution
Location: Odaiba, Tokyo, Japan
Opening: 1999

Renny wants to tweet how high she is above the ground every five minutes. She decides that she will have to figure out how high she will be beforehand. How high will Renny be at five minutes, 10 minutes, and 15 minutes? Provide a written explanation, along with your mathematical evidence, of why you took the steps that you did to find the three heights.

Standards: HSF.TF.B.5
Math Practices: 1, 2, 4, 6

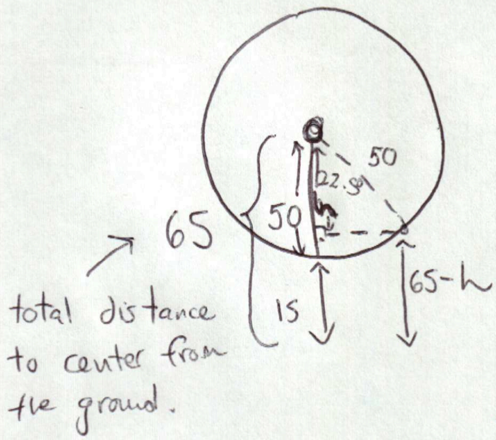
I first drew a circle to represent my ferris wheel



Since the ferris wheel is only 100 meters in diameter but it reaches its highest point at 115 I am going to assume it is 15 feet above the ground

After drawing my ferris wheel and labeling it I realized I had to calculate how much the wheel moved per minute since Renny wants to brag to her friends in minute intervals (5, 10, 15). So I set up a proportion and reduced it to 1 minute and changed the revolution to degrees.

$$\frac{16 \text{ minutes}}{1 \text{ revolution}} = \frac{16 \text{ minutes}}{360^\circ} = \frac{2 \text{ minutes}}{45^\circ} = \frac{1 \text{ minute}}{22.5^\circ}$$



65-h is the height of the carriage. To find h I drew a right triangle where 50, the radius, was my hypotenuse and h is my adjacent

$$\cos(22.5t) = \frac{h}{50}$$

Since I have adjacent and hypotenuse I can set up the above equation. I solve for h.

$$50\cos(22.5t) = h$$

Now I plug in for h for my expression of the height of the cart.

$$65 - 50\cos(22.5t)$$

I tested the ~~expres~~ function in my calculator setting the domain to [0,16] and it worked out. My highest point was 115 and my lowest point was 15.

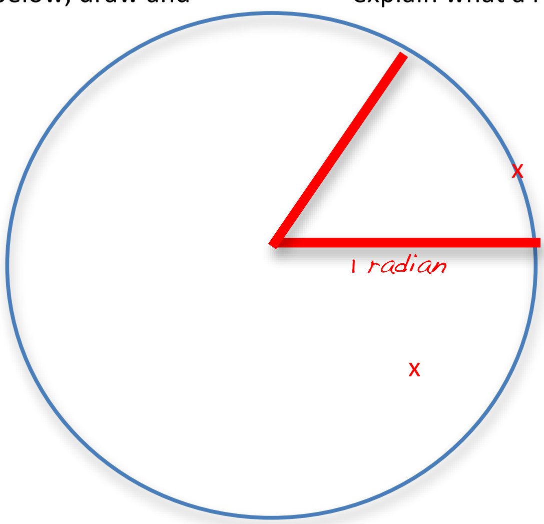
I plug the ~~pts~~ values 5, 10 and 15 and get

(5, 84) (10, 100), (15, 19)

I rounded because Renny seems to be the type that would only send whole numbers to her friends

What Is a Radian?

Using the circle below, draw and explain what a radian is.

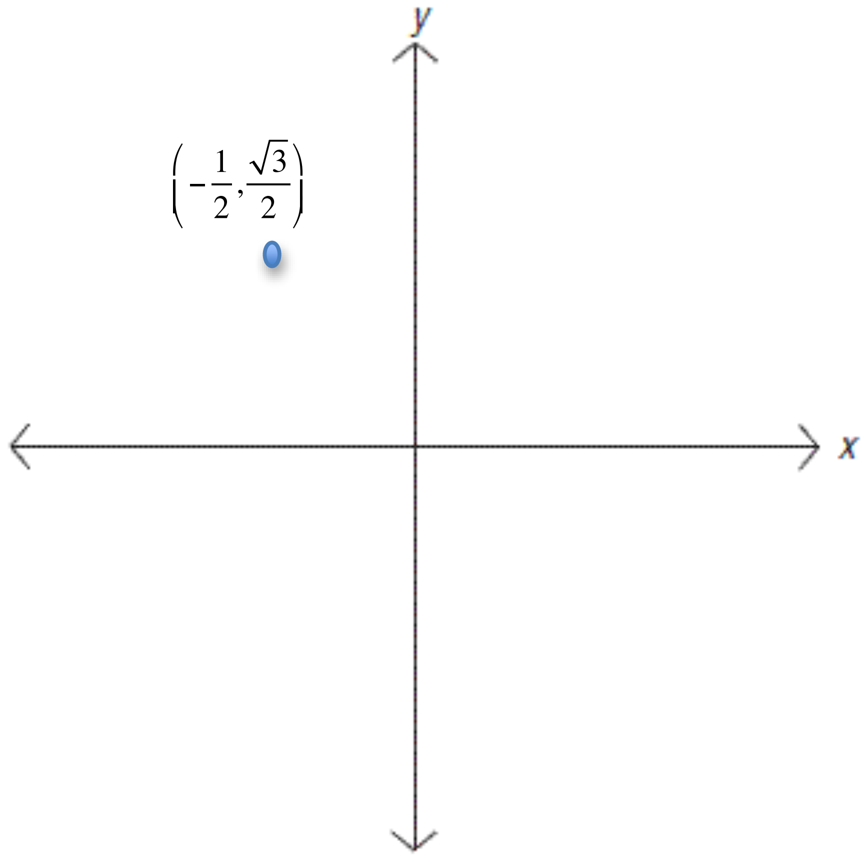


A radian is an angle measure for the angle at the center of a circle where the length of the arc made by the two radii is equal to the length of the radius.

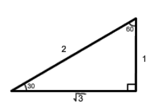
How long is the arc subtended by an angle of $\frac{4\pi}{3}$ radians on a circle of radius 15 cm?

$(4\pi/3)(15) = (4\pi)(5) = \boxed{20\pi \text{ cm}}$

Find the radian measure of the angle between the point on the coordinate plane below and *x*-axis:



Using the unit circle and the special 30-60-90

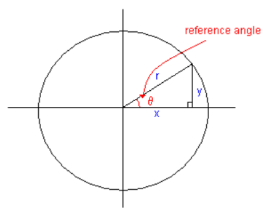


We first recognize that if the point was in the first quadrant, the sine of the angle is $\frac{\sqrt{3}}{2}$; therefore, the degree is 60, and so the angle between the point and the x-axis is 30 degrees, or π/6. Since the point is in the second quadrant (one quadrant over), we add π/2. π/2 + π/6 = 3π/6 + π/6 = 4π/6 = 2π/3. We change the radian measure to degree 2π/3 X 180/π = 120 degrees.

Pythagorean Identity

Explain why $\sin\Theta^2 + \cos\Theta^2 = 1$.

A triangle is drawn inside a unit circle so that its hypotenuse is the radius of the unit circle.



The radius of a unit circle is 1; therefore, the hypotenuse of the triangle is also 1. We use the definitions of the trigonometric ratios for sine and cosine to find the lengths of the sides of the triangle.

$\sin\Theta$ = opposite/hypotenuse → $\sin\Theta$ = opposite/1 → $\sin\Theta$ = opposite side

$\cos\Theta$ = adjacent/hypotenuse → $\cos\Theta$ = adjacent/1 → $\cos\Theta$ = adjacent side

Lastly, substitute the values of the side lengths and use the Pythagorean theorem since the triangle drawn is a right triangle.

$$(\sin\Theta)^2 + (\cos\Theta)^2 = 1^2$$

$$\sin\Theta^2 + \cos\Theta^2 = 1$$

Using the Pythagorean identity, find the value of sinθ and tanθ if $\cos\theta = -\frac{1}{2}$.

Substitute -½ for cosθ into the Pythagorean identity, $\sin^2+\cos^2 = 1$.

$\sin^2\theta + (½)^2 = 1$

$\sin^2\theta + ¼ = 1 \rightarrow \sin^2\theta + ¼ - (¼) = 1 - (¼) = ¾$

$\sin^2\theta = ¾ \rightarrow \sqrt{(\sin^2\theta)} = \sqrt{(¾)} \rightarrow \boxed{\sin\theta = \sqrt{3}/\sqrt{4} = \sqrt{3}/2}$

$$\frac{\sin^2}{\cos^2} + \frac{\cos^2}{\cos^2} = \frac{1}{\cos^2}$$

$$\tan^2 + 1 = \frac{1}{\left(-\frac{1}{2}\right)^2}$$

$$\tan^2 + 1 = 4$$

$$\tan^2 = 3$$

$$\tan = \sqrt{3}$$

GRADE 11 MATH: FERRIS WHEEL

INSTRUCTIONAL SUPPORTS

Unit Outline

INTRODUCTION: This unit outline provides an example of how to integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic. The length of the unit includes suggested time spent on the classroom instruction of lessons and administration of assessments. Please note that this framework does not include individual lessons.*

Grade Subject: Algebra II

UNIT TOPIC AND LENGTH:

- This unit should be covered in 10 instructional days (for periods of length of 47 minutes). Of the 10 days, two days should be set aside for assessment.

COMMON CORE STANDARDS:

- F-TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- F-TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- F-TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
- F-TF.C.8 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.

STANDARDS FOR MATHEMATICAL PRACTICE:

- MP.1 Make sense of problems and persevere in solving them.
- MP.2 Reason abstractly and quantitatively.
- MP.4 Model with mathematics.

BIG IDEAS/ENDURING UNDERSTANDINGS:

- Students will understand that a radian is unit of angle, equal to an angle at the center of a circle whose arc is equal in length to the radius.
- Students will understand that because the unit circle has a radius of 1, right triangles drawn

ESSENTIAL QUESTIONS:

- What is a radian?
- What are the benefits of using radians a measure of an angle instead of degrees?
- When can a situation be modeled using a trigonometric function?

inside the circle that have the hypotenuse equal to the length of the radius will have side lengths equal to cosΘ and sinΘ. ➢ Students will understand that periodic phenomenon can be described by a trigonometric function; once the midline, period and amplitude are identified.	➢ What is the unit circle? ➢ How can the unit circle be used in the coordinate plane to allow us to use trigonometric functions? ➢ How can we prove the Pythagorean identity? ➢ How can we manipulate the Pythagorean identity to find the tangent of an angle?
CONTENT: ➢ Students will learn how to use and prove the Pythagorean identity. ➢ Students will lean about the definition of the radian measure and how to use when calculating arc length or radius of a sector of a circle. ➢ Students will learn how to model periodic behavior with trigonometric functions. ➢ Students will learn how to use the unit circle to find angle measures (in both degrees and radians) for the angle between points on the coordinate plane and either the *x*- or *y*-axis.	**SKILLS:** ➢ If students are presented with two of three following values, they will be able to find the third: radius, angle measure in radians, and arc length. ➢ If students are a given a verbal description of a situation that can be modeled using trigonometric functions, they will identify the midline, amplitude, and frequency, and write the trigonometric equation of the function that models the situation. ➢ Students will be able to use the Pythagorean identity when finding the values of cosine, sine, and tangent.

VOCABULARY/KEY TERMS:

- RADIANS, ARC LENGTH, RADIUS, COSINE, TANGENT, SINE, PYTHAGOREAN IDENTITY, MIDLINE, FREQUENCY, AMPLITUDE

FORMATIVE ASSESSMENT: WHAT IS A RADIAN? (TF.A.1-2), PYTHAGOREAN IDENTITY (TF.C.8)

FINAL PERFORMANCE TASK: FERRIS WHEEL TASK (TF.B.5)

LEARNING PLAN AND ACTIVITIES:

Each *Arc of Lessons* and the student tasks included therein can be used in sequence for individual and group exploration and discussion, and for ongoing formative assessment. Each sequence of lessons here (an arc) includes specific student tasks. These tasks are arranged in a particular order to support the development of the big ideas of the unit.

Arc 1 (Understanding the Unit Circle): 5 days

- *Days 1-2:* Unit Circle activity with practice (https://www.khanacademy.org/math/trigonometry/less-basic-trigonometry/pythagorean-identity/e/pythagorean_identities)

- *Days 3-4:* Radians formative task with practice (https://www.khanacademy.org/math/trigonometry/basic-trigonometry/radians_tutorial/e/cc-radians-and-arc-length)

- *Day 5:* Formative assessment (What is a Radian? and Pythagorean Identity)

Arc 2 (Modeling with Trigonometric Functions): 5 days

- *Days 6-7:* Tire task and instruction/discussion

- *Days 8-10:* Ferris Wheel culminating task

Additional Support Strategies: Students who struggle with precision would benefit from a checklist, a word bank, or worked examples. Students who struggle with explaining the Pythagorean identity would benefit from having a worked example of the proof of the Pythagorean identity, using a proof-starter check list. For examples of checklists and sentence starters, visit the following documents on the Common Core Library, under the "See Student Work" tab: ELA/Literacy for ELLs, Mathematics for ELLs, and Students with Disabilities.

For students who struggle with visualizing the Tire task scenario, provide the interactive animation at http://www.absorblearning.com/media/attachment.action?quick=je&att=1388. As a support throughout the Ferris Wheel task, provide the interactive animation at http://illuminations.nctm.org/Activity.aspx?id=3589, where students can explore the amplitude, period, and phase shift by examining various trigonometric graphs. The resource section below provides videos for students to have additional examples and instruction.

A scaffolded version of the Ferris Wheel task (version B) is available for students who struggle with entering the original task, or for educators to provide for specific students. Teachers may also decide to use the prompts included in version B one by one, as student need for them arises.

Additional activities are available at http:///www.illustrativemathematics.org, if additional instruction is necessary for any of the included content, or for extension problems for students working above grade level. The following tasks are suggested:

- Properties of Trigonometric Functions
- Trigonometric Functions for Arbitrary Angles
- Trigonometric Ratios and the Pythagorean Theorem
- Fox and Rabbits 2
- Fox and Rabbits 3
- As the Wheel Turns

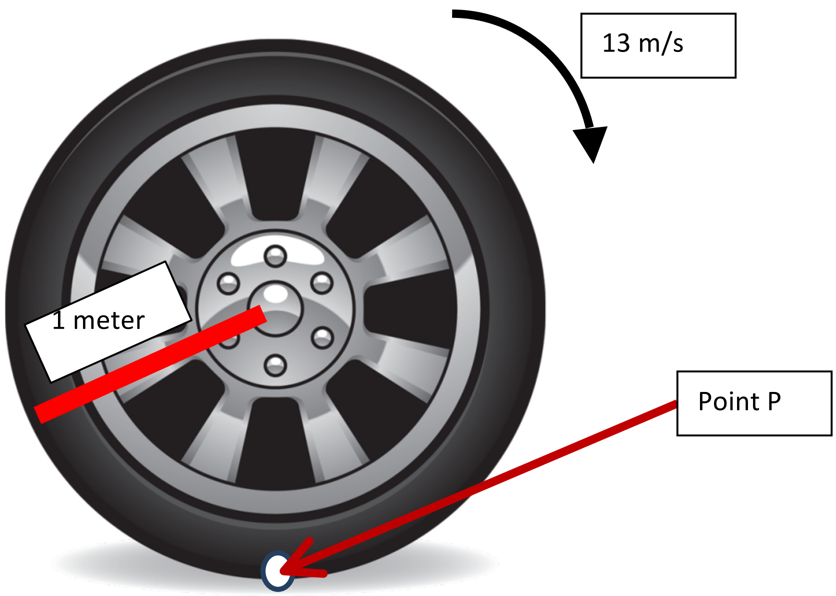
RESOURCES:

- https://www.khanacademy.org/math/trigonometry/basic-trigonometry
- https://www.khanacademy.org/math/trigonometry/basic-trigonometry/unit_circle_tut/v/ferris-wheel-trig-problem
- https://www.khanacademy.org/math/trigonometry/basic-trigonometry/unit_circle_tut/v/ferris-wheel-trig-problem--part-2

HS-F-TF.B.5

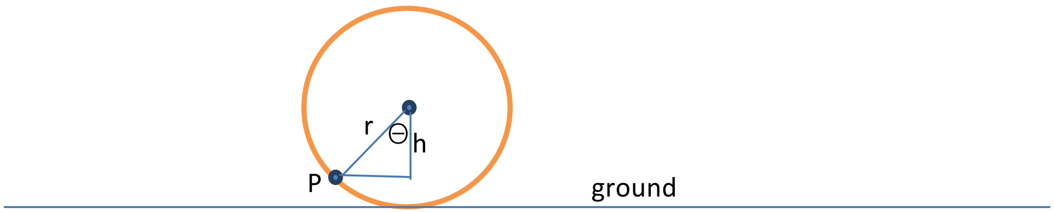
Tire Task

A tire of radius 1 meter begins to move along a flat surface so that the center of the tire moves forward at a constant speed of 13 meters per second. At the moment the tire begins to turn, a marked point P on the tire is touching the flat surface.



a) Calculate the rate at which wheel rotates in degrees (or radians) per second.

b) The diagram below is a sketch of point P's position after one second. It is labeled, and *r* represents the radius, ⊖ the angle after one second, and *h* the height of a triangle drawn between point P and the center of the wheel. Write an algebraic expression for the function *y* that gives the height (in meters) of the point P, measured from the flat surface, as a function of *t*, the number of seconds after the wheel begins moving.



c) Using the function, calculate point P's height at 1 second, 1 minute, and 1 hour.

d) We define the horizontal position of the point P to be the number of meters the point has traveled forward from its starting position, disregarding any vertical movement the point has made. Write an algebraic expression for the function x that gives the horizontal position (in meters) of the point P as a function of t, the number of seconds after the wheel begins moving.

e) Your classmate thinks it's silly to write the horizontal position of P as a trigonometric function. She thinks it would be better to just write it as a linear function, $h(x) = 13x$, where $h(x)$ is the horizontal distance traveled by point P and x is the time in seconds traveled. Why do you think she would say that? If the wheel was moving slower (2 m/s), would your friend change her mind? Why or why not?

Radians

Part 1: Discover

1. Fold a paper plate into *quarters*. Draw a radius from the center to the circumference of the circle. Label angle measurements in *90-degree increments* around the circle.
2. Cut a string the *length of one radius*. Start at 0^0 and mark “1 radius” around the circumference. From “1 radius,” mark another radius length and label it “2.” Continue until six radius lengths have been measured.
3. Create a dotted line from “1 radius” to the center. We could measure the angle between 0 and 1 with a protractor in degrees and get approximately 57^0, or we could call it **1 radian**.
4. What is the measure in radians of the angle created from the center between 0 and 2? 0 and 4? 0 and 6?

5. In a complete sentence, write your own definition of a radian or radian measure of angles.

6. About how many radians is a straight line? What is a mathematical value that is close to this number?

7. Define the relationship between radians and degrees: __________ radians = __________ degrees.

8. An angle's measurement in radians is numerically equal to the length of a corresponding arc of a unit circle (explore this at http://en.wikipedia.org/wiki/Radian). Can you make a connection to the circumference formula?

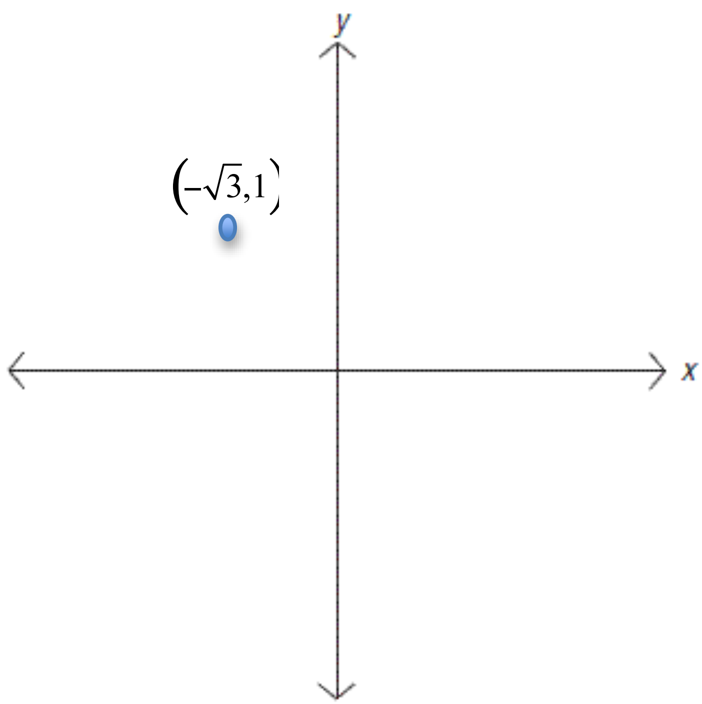
9. Label the measurements around your circle in radians, in addition to the degree measurements, and complete the circle.

Part 2: Analyze

1. In a circle, a central angle of ¼ radians subtends an arc of 2 centimeters. Find the length, in centimeters, of the radius of the circle.

2. Find, in centimeters, the length of an arc intercepted by a central angle of 5 radians in a circle with a radius of 2 centimeters.

3. Find the angle of the point below.

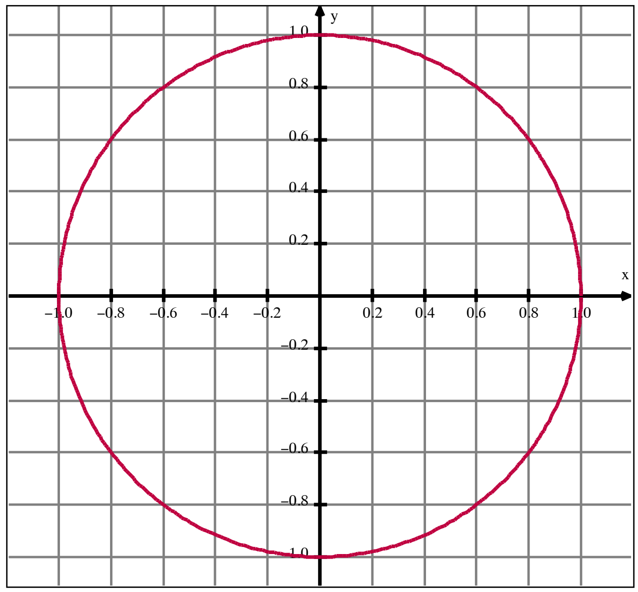


4. Using the Pythagorean identity, find the value of cosθ if $\sin\theta = -\frac{1}{2}$.

Standard: F-TF.A.2, F-TF.C.8 **Unit Circle Activity**

Part 1: Discover

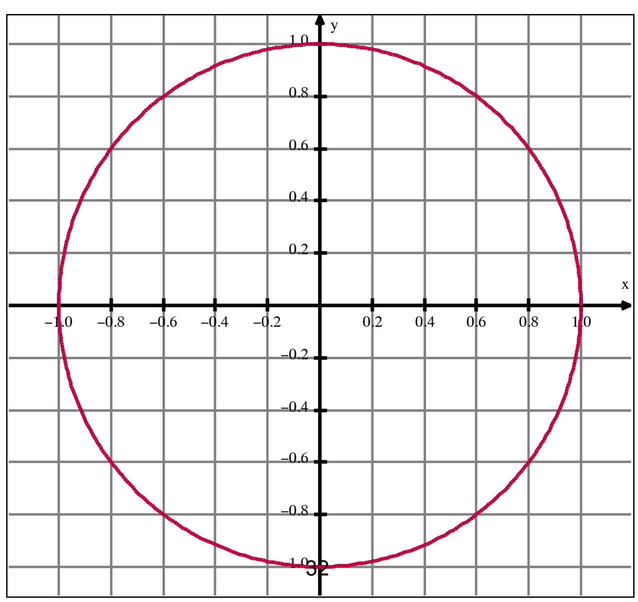
1. Below is a picture of the unit circle. Using the origin as your vertex, construct a 30-degree angle in Quadrant 1. Label the point where the angle intersects the unit circle with A. Identify the coordinates of point A and label it on the unit circle.



2. Construct a perpendicular segment from point A to the *x*-axis to form a right triangle. Determine the length of the hypotenuse and label it on the unit circle.

3. Using the trigonometric ratios sine and cosine, determine the lengths of the two legs of the triangle and label them on the unit circle. How do these lengths relate to the coordinates of A?

4. Using the origin as your vertex on the unit circle below, construct a 45-degree angle in Quadrant 1. Label the point where the angle intersects the unit circle with B. Identify the coordinates of point B and label it on the unit circle.

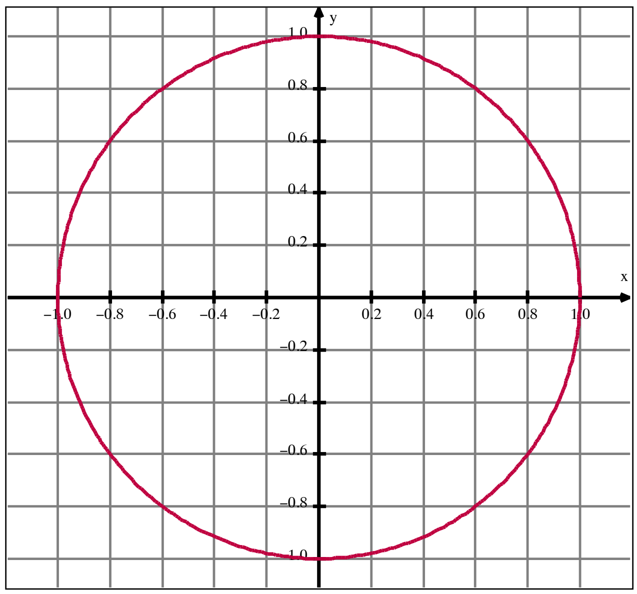


Unit Circle Activity

5. Construct a perpendicular segment from point B to the *x*-axis to form a right triangle. Determine the length of the hypotenuse and label it on the unit circle.

6. Using the trigonometric ratios sine and cosine, determine the lengths of the two legs of the triangle and label them on the unit circle. How do these lengths relate to the coordinates of B?

7. Using the origin as your vertex on the unit circle below, construct a 60-degree angle in Quadrant 1. Label the point where the angle intersects the unit circle with C. Identify the coordinates of point C and label it on the unit circle.



8. Construct a perpendicular segment from point C to the *x*-axis to form a right triangle. Determine the length of the hypotenuse and label it on the unit circle.

9. Using the trigonometric ratios sine and cosine, determine the lengths of the two legs of the triangle and label them on the unit circle. How do these lengths relate to the coordinates of C?

Part 2: Analyze

1. Complete the following table for the angles constructed in Part 1.

θ	*x*-coordinate	*y*-coordinate

Unit Circle Activity

2. Which coordinate correlates with the horizontal leg of the right triangle? Which coordinate correlates with the vertical leg of the right triangle? What is the length of the hypotenuse? Is this true for all angles on the unit circle? Why?

3. There are a few angles we do not draw right triangles for because they are angles with terminal sides on the axes. What are these angles? What are their coordinates on the unit circle?

θ	*x*-coordinate	*y*-coordinate

4. The Pythagorean theorem states $a^2 + b^2 = c^2$. Substitute the corresponding values from Part 2, Question 3, into the Pythagorean theorem. Does this hold true for the right triangles created in Part 1?

   a. Make and justify your prediction.

   b. Test your prediction.

   c. What can you conclude? Was your prediction correct?