

GRADE 5 MATH: STUFFED WITH PIZZA

UNIT OVERVIEW

In this unit students will develop and expand the concept of rational numbers by using several interpretations and different types of physical models.

TASK DETAILS

Task Name: Stuffed with Pizza

Grade: 5

Subject: Mathematics

Depth of Knowledge: 2

Task Description: Students use fractional parts of a whole, addition and subtraction of fractions, and comparison, to determine if two boys eat the same amount or a different amount of pizza pieces.

Standards:

5.NF.1 Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $\frac{2}{3} + \frac{5}{4} = \frac{8}{12} + \frac{15}{12} = \frac{23}{12}$. (In general, $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$.)

5.NF.2 Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result $\frac{2}{5} + \frac{1}{2} = \frac{3}{7}$, by observing that $\frac{3}{7} < \frac{1}{2}$.

Standards for Mathematical Practice:

MP.1 Make sense of problems and persevere in solving them.

MP.3 Construct viable arguments and critique the reasoning of others.

MP.6 Attend to precision.

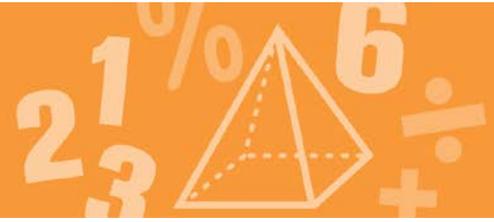


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The task and instructional supports in the following pages are designed to help educators understand and implement tasks that are embedded in Common Core-aligned curricula. While the focus for the 2011-2012 Instructional Expectations is on engaging students in Common Core-aligned culminating tasks, it is imperative that the tasks are embedded in units of study that are also aligned to the new standards. Rather than asking teachers to introduce a task into the semester without context, this work is intended to encourage analysis of student and teacher work to understand what alignment looks like. We have learned through this year’s Common Core pilots that beginning with rigorous assessments drives significant shifts in curriculum and pedagogy. Universal Design for Learning (UDL) support is included to ensure multiple entry points for all learners, including students with disabilities and English language learners.

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Acknowledgements: The unit outline was developed by Shenaz Hashim (CFN 109), Haydee Santino and Magaly De La Cruz with input from Curriculum Designers Alignment Review Team. The tasks were developed by the schools in the 2010-2011 NYC DOE Elementary School Performance Based Assessment Pilot, in collaboration with Exemplars, Inc. and Center for Assessment.



GRADE 5 MATH: STUFFED WITH PIZZA

PERFORMANCE TASK

This section includes the performance task and a planning sheet which highlights the underlying mathematical concepts, problem-solving strategies and mathematical language associated with the task. Potential solution paths and connections that students may make while completing the task are also identified.

Name. _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Stuffed with Pizza Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link Everyday Mathematics, Unit 5
 Common Core Standard(s) _____

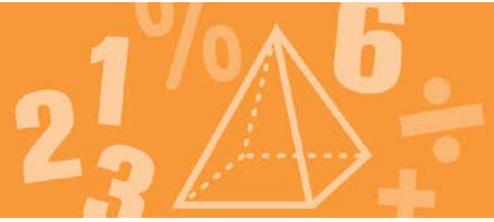
Underlying Mathematical Concepts	Problem Solving Strategies/Representation	Mathematical Language
fractional parts of a whole fraction notation fraction addition comparison of fractions with unlike denominators improper proper fractions	model (manipulatives/fraction bars) diagram/key - area model/ circle graph chart	model whole area model equivalent circle graph rectangular diagram per key greater/less more/less than than fraction $\frac{2}{8}, \frac{3}{8} \dots$ $>, <$ percents % decimals numerator denominator Related Tasks
Possible Solution(s) pizzas	Connections <ul style="list-style-type: none"> $\frac{1}{8}$ cheese is eaten $\frac{3}{8}$ pepperoni eaten 1 whole mushroom eaten It appears that Luis likes cheese the most and pepperoni the least. It appears that Tito likes mushroom more than cheese $\frac{1}{2}$ pizza is 50% or .5 $\frac{1}{4}$ pizza is 25% or .25 $\frac{1}{2} = \frac{4}{8}$ There is $\frac{6}{8}$ slices left but not from 1 pizza 	Sec Resource Binder <ul style="list-style-type: none"> 100% mushroom eaten Relate to a similar problem and state math If use rectangular pizzas the amount per boy is the same $\frac{5}{8}$ pepperoni left which is greater than 50%, $\frac{1}{2}$
<p>cheese</p> <p>Tito $\frac{2}{8} + \frac{3}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$</p> <p>Luis $\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$</p> <p>pepperoni</p> <p>mushroom</p>	<p>Answer Luis is correct</p> <p>key T Tito L Luis</p>	

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Stuffed with Pizza

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GRADE 5 MATH: STUFFED WITH PIZZA
UNIVERSAL DESIGN FOR LEARNING (UDL)
PRINCIPLES

Math Grade 5 - Stuffed With Pizza Common Core Learning Standards/ Universal Design for Learning

The goal of using Common Core Learning Standards (CCLS) is to provide the highest academic standards to all of our students. Universal Design for Learning (UDL) is a set of principles that provides teachers with a structure to develop their instruction to meet the needs of a diversity of learners. UDL is a research-based framework that suggests each student learns in a unique manner. A one-size-fits-all approach is not effective to meet the diverse range of learners in our schools. By creating options for how instruction is presented, how students express their ideas, and how teachers can engage students in their learning, instruction can be customized and adjusted to meet individual student needs. In this manner, we can support our students to succeed in the CCLS.

Below are some ideas of how this Common Core Task is aligned with the three principles of UDL; providing options in representation, action/expression, and engagement. As UDL calls for multiple options, the possible list is endless. Please use this as a starting point. Think about your own group of students and assess whether these are options you can use.

REPRESENTATION: *The “what” of learning.* How does the task present information and content in different ways? How students gather facts and categorize what they see, hear, and read. How are they identifying letters, words, or an author's style?

In this task, teachers can...

- ✓ **Provide multiple entry points to a lesson and optional pathways through content (e.g., exploring big ideas through dramatic works, arts and literature, film and media)** through the exploration of the understanding of basic fractions, equivalent fractions, and addition of fractions.

ACTION/EXPRESSION: *The “how” of learning.* How does the task differentiate the ways that students can express what they know? How do they plan and perform tasks? How do students organize and express their ideas?

In this task, teachers can...

- ✓ **Provide graphic organizers and templates for data collection and organizing information** in order to provide a tool to manage figures and calculations.

ENGAGEMENT: *The “why” of learning.* How does the task stimulate interest and motivation for learning? How do students get engaged? How are they challenged, excited, or interested?

In this task, teachers can...

- ✓ **Prompt or require learners to explicitly formulate or restate goal** by having students work in pairs to summarize and define the steps to solving the problem.

Visit <http://schools.nyc.gov/Academics/CommonCoreLibrary/default.htm> to learn more information about UDL.



GRADE 5 MATH: STUFFED WITH PIZZA

RUBRIC

The following section contains two rubrics that were used to score student work: a content rubric and a process rubric. The content rubric describes student performance according to the content standards in the CCLS. The process rubric describes student performance according to the National Council of Teachers of Mathematics (NCTM) process standards. Students were given a score based on their achievement on the CCLS content rubric and the process rubric. Given that the process rubric is not in the language of the Common Core’s Mathematical Practices, we have also included a document that NCTM has posted regarding the relationship between the NCTM process standards and the Standards for Mathematical Practice.

CCSS Mathematics Content Standards Rubric – Grade 5

Students apply mathematical concepts, reasoning, and procedural skills in problems-solving situations and support their solutions using computations, mathematical language, and appropriate representations/ modeling.

CCSS Math Criteria by Strand	Novice	Apprentice	Practitioner	Expert
Number & Operations in Base Ten & Number & Operations - Fractions	<p>Consistently flawed understanding of:</p> <ul style="list-style-type: none"> • equivalent fractions and mixed numbers • decimal values • place value <p>Fractional or decimal representations incorrect or not appropriate for task</p> <p>Incorrect computational strategies used or major inaccuracies in computation lead to an incorrect solution</p> <p>A correct answer may be stated, but is not supported by student work or explanations</p>	<p>Some parts of problem correct and those parts supported by student work (e.g., uses visual models to represent fractional or decimal parts of a whole)</p> <p>Mostly consistent and accurate understanding of:</p> <ul style="list-style-type: none"> • equivalent fractions and mixed numbers • representation of fractional notation • rounding whole numbers and decimals using place value <p>Uses additive reasoning to solve or interpret most problems</p> <p>May include limited Explanations for solutions</p> <p>Displays some inaccuracies in computation (e.g., multiplying multi-digit whole numbers using the standard algorithm)</p>	<p><i>Uses clear and consistent fractional and decimal representations (e.g., using visual models- number line, area, sets; symbols, expanded form) when reading, writing, and comparing quantities</i> 5.NF-3, 5 5.NBT-3</p> <p><i>Uses addition and subtraction to solve problems with fractions with unlike denominators</i> 5.NF-1, 2</p> <p><i>May be some minor flaws when performing multi-step computations involving addition, subtraction, multiplication, or division (with fractions, mixed numbers, whole numbers, or decimals), <u>but procedural and conceptual understanding is clearly evident</u></i> 5.NBT-6, 7 5.NF-1, 2, 6, 7</p>	<p>All parts of problem correct, precise, and supported by student work or explanations</p> <p>Demonstrates deeper understanding of fractions, whole numbers, and decimals by relating them to percents or other abstract concepts beyond the scope of a specific task (e.g., verifying the solution or approach using alternative models or equations; making and supporting reasonable estimates in multi-step problems)</p> <p>Uses a variety of strategies and four operations to solve problems with whole numbers, decimals, fractions (including mixed numbers)</p> <p>Consistently applies multiplicative reasoning when appropriate</p>
Operations & Algebraic Thinking	<p>Identifies numeric patterns (e.g., multiplying or dividing by powers of 10) or describes differences in numeric patterns (repeating or growing)</p>	<p>Writes a numerical expression for a given situation using one or more of the four operations and some symbols (parentheses, brackets, or braces)</p> <p>Extends a numeric pattern when given a rule (e.g., creates ordered pairs or completes input-output table)</p> <p>Graphs ordered pairs on a coordinate grid</p>	<p><i>Writes, interprets, and evaluates numerical expressions</i> 5.OA-1, 2</p> <p><i>Analyzes patterns and numeric relationships using tables, graphs, words, or symbolic rules (e.g., generalizes a pattern to create a rule; forms ordered pairs; explains place value patterns)</i> 5.OA-3; 5.NB-1, 2</p>	<p>Generates multiple representations when solving problems (e.g., to show reasonableness of the answer or an alternative approach or solution)</p> <p>Explains or describes functions as linear or nonlinear when represented in graphical or tabular representations</p>

NOTE: Anchor papers will illustrate how descriptors for each performance level are evidenced at each grade.

Exemplars[®] Standards-Based Math Rubric*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	<p>No strategy is chosen, or a strategy is chosen that will not lead to a solution.</p> <p>Little or no evidence of engagement in the task present.</p>	<p>Arguments are made with no mathematical basis.</p> <p>No correct reasoning nor justification for reasoning is present.</p>	<p>No awareness of audience or purpose is communicated.</p> <p style="text-align: center;">or</p> <p>Little or no communication of an approach is evident</p> <p style="text-align: center;">or</p> <p>Everyday, familiar language is used to communicate ideas.</p>	<p>No connections are made.</p>	<p>No attempt is made to construct mathematical representations.</p>
Apprentice	<p>A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen.</p> <p>Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.</p>	<p>Arguments are made with some mathematical basis.</p> <p>Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.</p>	<p>Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task.</p> <p style="text-align: center;">or</p> <p>Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols.</p> <p style="text-align: center;">or</p> <p>Some formal math language is used, and examples are provided to communicate ideas.</p>	<p>Some attempt to relate the task to other subjects or to own interests and experiences is made.</p>	<p>An attempt is made to construct mathematical representations to record and communicate problem solving.</p>

*Based on revised NCTM standards.

Exemplars[®] Standards-Based Math Rubric (conf.)*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Practitioner	<p>A correct strategy is chosen based on mathematical situation in the task.</p> <p>Planning or monitoring of strategy is evident.</p> <p>Evidence of solidifying prior knowledge and applying it to the problem solving situation is present.</p> <p>Note: The practitioner must achieve a correct answer.</p>	<p>Arguments are constructed with adequate mathematical basis.</p> <p>A systematic approach and/or justification of correct reasoning is present. This may lead to...</p> <ul style="list-style-type: none"> • clarification of the task. • exploration of mathematical phenomenon. • noting patterns, structures and regularities. 	<p>A sense of audience or purpose is communicated.</p> <p style="text-align: center;">and/or</p> <p>Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response.</p> <p>Formal math language is used throughout the solution to share and clarify ideas.</p>	<p>Mathematical connections or observations are recognized.</p>	<p>Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.</p>
Expert Work at this level is exceeding grade-level expectations	<p>An efficient strategy is chosen and progress towards a solution is evaluated.</p> <p>Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered.</p> <p>Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present.</p> <p>Note: The expert must achieve a correct answer.</p>	<p>Deductive arguments are used to justify decisions and may result in formal proofs.</p> <p>Evidence is used to justify and support decisions made and conclusions reached. This may lead to...</p> <ul style="list-style-type: none"> • testing and accepting or rejecting of a hypothesis or conjecture. • explanation of phenomenon. • generalizing and extending the solution to other cases. 	<p>A sense of audience and purpose is communicated.</p> <p style="text-align: center;">and/or</p> <p>Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties.</p> <p>Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.</p>	<p>Mathematical connections or observations are used to extend the solution.</p>	<p>Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.</p>

*Based on revised NCTM standards.

National Council of Teachers of Mathematics (NCTM) Process Standards and the Common Core State Standards for Mathematics

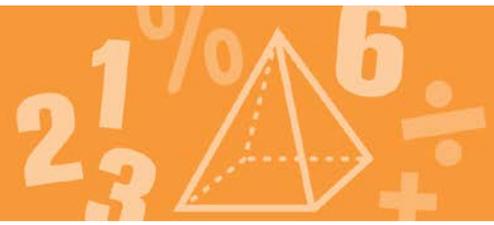
From *NCTM Action on the Common Core State Standards for Mathematics*

by NCTM President J. Michael Shaughnessy

The preeminent message in both the NCTM *Principles and Standards for School Mathematics* (2000) and CCSSM is the importance of nurturing mathematical thinking and reasoning processes in our students. No bulleted list of specific content standards will hold together as a coherent, meaningful whole, or make any significant contribution to our students' growth in mathematics, without interweaving mathematical "practices." Mathematics curricula must show students the power of reasoning and sense making as they explore mathematical structures, of communication as they construct viable arguments, and of multiple representations as they engage in mathematical modeling. The close connections between the NCTM Process Standards and the CCSSM Standards for Mathematical Practice are represented in the chart below.

The upcoming NCTM publication, *Making it Happen*, will provide a deeper analysis of the connections between the NCTM Process Standards and detail the potential of the existing NCTM resources to interpret and implement CCSSM.

NCTM Process Standards and the CCSS Mathematical Practices	
NCTM Process Standards	CCSS Standards for Mathematical Practice
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Communication	3. Construct viable arguments
Connections	6. Attend to precision. 7. Look for and make use of structure
Representations	4. Model with mathematics.



GRADE 5 MATH: STUFFED WITH PIZZA

ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points, student summaries, and implications for instruction for each performance level. The annotated student work and student summaries demonstrate performance at different levels and show examples of student understandings and misunderstandings of the task that can be used with the implications for instruction to understand how to move students to the next performance level.

Grade 5 Math: Stuffed with Pizza Annotated Student Work: Expert

P/S	R/P	Com.	Con.	Rep.	A/L
E	E	E	E	E	E

Expert, Student 1

Grade 5

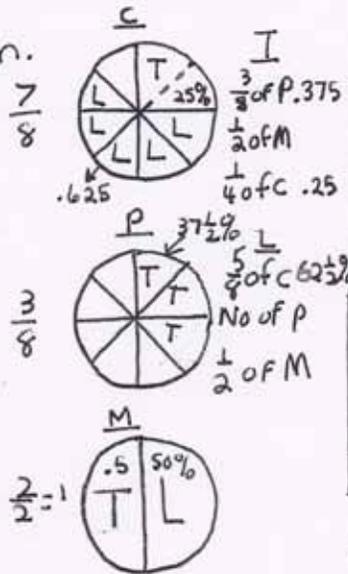
Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

I will find who is correct, Tito or Luis.
I will make a diagram.

Key	
T	TITO
L	Luis
C	cheese
P	Pepperoni
M	Mushroom
↑ pizzas	



Tito ate

$$\frac{3}{8} + \frac{1}{2} + \frac{1}{4} = ?$$

$$\frac{3}{8} + \frac{4}{8} + \frac{2}{8} = \frac{9}{8} = \boxed{1\frac{1}{8}}$$

Luis ate

$$\frac{5}{8} + \frac{1}{2} = ?$$

$$\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = \boxed{1\frac{1}{8}}$$

you have to find how to have 8 in the denominator so you add equivalent fractions

on to connections

Answer: Luis was right because they both ate $1\frac{1}{8}$ pizza

The student is able to make sense and persevere in solving the problem. The student demonstrates correct reasoning of proportional parts of a whole, correctly assigns each boy pizza pieces, and finds the correct equivalent fractions to state a correct answer. The student verifies her/his answer with decimals and percents and brings prior knowledge of statistics to the solution.

The student models with mathematics. The area model/diagram of the pizzas is accurate, labeled, and a key defines Tito, Luis, and the types of pizzas. The student uses the diagram to record some of her/his extended thinking to percents and decimals.

The student uses precise mathematical terms: amount, diagram key, denominator, equivalent fractions, most, ratio, total, mode, minimum, most likely, least, whole, percents, decimals, as well as fraction, decimal, and percent notation to help construct a viable argument.

Grade 5 Math: Stuffed with Pizza Annotated Student Work: Expert

The student looks for and makes use of the structures in the problem. The student makes a number of connections in her/his solution and also verifies her/his answer by using percents and decimals to represent the fractions used in the diagram. The student states, "I know I am right that they are the same total amount."

Connections — **Follow the dots**

- They ate $2\frac{2}{8} = 2\frac{1}{4}$ pizza in all
- $\frac{6}{8}$ of a pizza is left or $\frac{3}{4}$
- Luis eats the most of 1 pizza - the C one
- I can do ratio. If there are 24 total slices then Tito eats $\frac{3}{24}$ P

I will use my key for C, P, M

4 out of 24 $\frac{1}{6}$ M $\frac{3}{8}$ C L eats $\frac{5}{24}$ C $\frac{2}{24}$ P $\frac{4}{24}$ M

- It is most likely Tito likes M pizza the most and P pizza the least
- $\frac{1}{2}$ M is 50% of a pizza so is $\frac{1}{2}$ L That is .5
- $\frac{5}{8}$ C is $8\overline{)5.000}$ That is $62\frac{1}{2}\%$

Luis's model is C
Tito's minimum is C

- $\frac{1}{4}$ C = .25 That is 25%
- $\frac{3}{8}$ P is $8\overline{)3.000}$ That is $37\frac{1}{2}\%$
- $\frac{1}{8}$ is $8\overline{)1.000}$ or $12\frac{1}{2}\%$

Tito eats 12.5
12.5
 $+12.5$
 37.5 or $37\frac{1}{2}\%$
of P pizza

- I can verify with multiplication $\frac{1}{8} = 12\frac{1}{2}\%$ or 12.5%

Tito eats $1\frac{1}{8}$ 1.000 - whole
 $+ .125$ - $\frac{1}{8}$
 1.125

Luis eat $1\frac{1}{8}$ 1.00 - whole
 $+ .125$ - $\frac{1}{8}$
 1.125

I know it is right that they ate the same total amount

- I can verify with percents

Luis C 62.5% M 50.0% <hr/> 112.5%	Tito C 25.0% P 37.5% M 50.0% <hr/> 112.5%
---	---

-same again
I like using decimals and percents this way
page 2 of 2

12.5
 $\times 2$

 25.0

12.5
 $\times 3$

 37.5

The end

The student shows evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student solves a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

**Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Expert**

Expert – Student 1 Summary

Achievement Level: Student 1 is an Expert according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Expert	The student’s strategy of using a diagram to determine the fractional part of each pizza Tito and Luis eats, finding the fractional total amount of slices eaten, and comparing the totals to determine who is correct, works to solve the problem. The student’s answer, “Luis was correct because they both ate $1 \frac{1}{8}$ pizza,” is correct. The student verifies her/his answer with decimals and percents to determine if the fractional values in her/his diagram are correct. The student also brings prior knowledge of statistics to her/his solution.
Reasoning and Proof Expert	The student demonstrates correct reasoning of proportional parts of a whole. The student correctly adds the eighths of pizza Tito and Luis eat for a total of $\frac{9}{8} = 1 \frac{1}{8}$ and correctly reasons that Luis is correct. The student justifies her/his answer and explains the phenomenon of fractions by linking them to decimals and percents and to verify that the fractions indicated in her/his solution are correct.
Communication Expert	The student correctly uses the mathematical term-amount, from the problem. The student also correctly uses the terms-diagram, key, denominator, equivalent fractions, most, ratio, 4 out of 24, 2 out of 24, 5 out of 24, 0 out of 24, 3 out of 24, total, mode, minimum, most likely, least, whole, percents, decimals. The student correctly uses the mathematical notation- $\frac{7}{8}$, $\frac{3}{5}$, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{4}{8}$, $\frac{9}{8}$ $1 \frac{1}{8}$, $2 \frac{2}{8}$, $2 \frac{1}{4}$, $\frac{6}{8}$, $\frac{3}{4}$, 50%, $62 \frac{1}{2} \%$, 25%, $37 \frac{1}{2} \%$, $12 \frac{1}{2} \%$, 12.5 %, 62.5%, 50.0%, 25.0 5, 37.5%, 50.0%, 112.5%, .5, .625, .25, .375, .125, 1.125, 100.0.

<p>Connections Expert</p>	<p>The student makes the Practitioner connections, "They ate $2\frac{2}{8} = 2\frac{1}{4}$ pizza in all," "$\frac{6}{8}$ of pizza is left or $\frac{3}{4}$," "Luis eats the most of 1 pizza-the C one." The student makes Expert connections by extending her/his thinking to other mathematical content/standards. The student states, "I can do ratio. If there are 24 total slices then Tito eat $\frac{3}{24}$ P, $\frac{4}{24}$, M $\frac{2}{24}$ C," "L eats $\frac{5}{24}$ C, $\frac{0}{24}$ P, $\frac{4}{24}$ M," "It is most likely Tito likes M pizza the most and P pizza the least," "Luis's mode is C," "Tito's minimum is C." The student verifies his/her diagram by using percents and decimals to represent the fractions used in the diagram-$\frac{1}{2}$, $\frac{5}{8}$, $\frac{1}{4}$, $\frac{3}{8}$, $\frac{1}{8}$. The student states, "I know it is right that they ate the same total amount," and, "same again."</p>
<p>Representation Expert</p>	<p>The student's diagram is appropriate to the problem and accurate. A key defines the labels and entered data and the circles are correctly proportioned. The student uses her/his diagram to record some of her/his extended thinking to percents and decimals.</p>

Instructional Implications: Stuffed with Pizza, 5

Achievement Level: Expert

Note: Student work identified at this level is exceeding grade-level expectations

The following is a list of instructional implications that you may want to consider for students performing at the Expert level. In addition, you may want to consult the suggestions for the Practitioner level:

- Solve problem more than one way to verify that the answer is correct and link the two strategies together
- Relate problem to a similar one completed and discuss how they are mathematically similar
- Use percents to define the number of pizza pieces each boy eats
- Use decimals to define the number of pizza pieces each boy eats
- Use ratio to define the number of pizza pieces each boy eats
- Bring the content of statistics to the problem-most likely, minimum, maximum, mode

Grade 5 Math: Stuffed with Pizza

Annotated Student Work: Practitioner

Practitioner, Student 1
Name _____

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Stuffed with Pizza

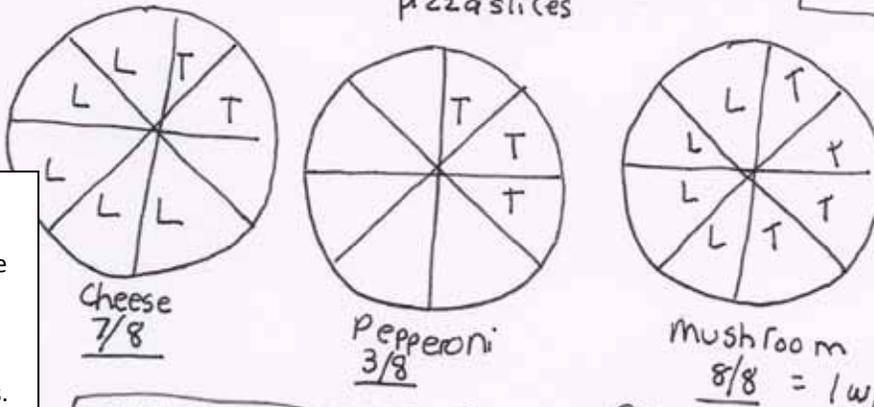
Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

The student is able to make sense and persevere in solving the problem. The student demonstrates correct reasoning by assigning each boy pizza pieces, and finds the correct equivalent fractions to state a correct answer.

I have to find out who is correct
I will make a diagram
pizza slices



The student models with mathematics. The area model/diagram of the pizzas are accurate, labeled, and a key defines Tito and Luis.



The student uses precise mathematical terms-amount, diagram, key, and fraction notation to help construct a viable argument.

The student shows evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student solves a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

$$T = \frac{2}{8} + \frac{3}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$$

$$L = \frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$$

My Answer
Luis is right. They ate the same amount of pizza

- Connections
- They both ate a total of $2\frac{1}{8} = 2\frac{1}{4}$ pizzas
 - $\frac{6}{8}$ of the pizza is left
 - Luis ate the most of 1 type of pizza - cheese
 - Luis and Tito ate an equal amount of mushroom pizza

The student looks for and makes use of the structures in the problem. The student makes a list of four connections s/he sees in her/his solution.

page 1 of 1

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Practitioner

Practitioner – Student 1 Summary

Achievement Level: Student 1 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student’s strategy of diagramming a cheese pizza, a pepperoni pizza, a mushroom pizza, labeling each fractional part of the pizza eaten, finding the total amount each boy ate and comparing those totals, works to solve the problem. The student’s answer, “Luis is right. They ate the same amount of pizza,” is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of proportional parts of a whole. Each pizza is proportioned in eighths and the pieces each boy eats are labeled. The student correctly finds the correct equivalent fractions in eighths for one-half, and one-fourth. The student correctly adds the fractions and compares the sum to determine that Luis was correct.
Communication Practitioner	The student correctly uses the mathematical term-amount, from the task. The student also correctly uses the terms-diagram, key. The student correctly uses the mathematical notation- $\frac{7}{8}$, $\frac{3}{8}$, $\frac{8}{8}$, $\frac{2}{8}$, $\frac{4}{8}$, $\frac{9}{8}$, $1\frac{1}{8}$, $\frac{5}{8}$, $2\frac{2}{8}$, $2\frac{1}{4}$, $\frac{6}{8}$.
Connections Practitioner	The student solves the problem and makes the mathematically relevant observations, “They both ate a totle of $2\frac{2}{8} = 2\frac{1}{4}$ pizzas,” “ $\frac{6}{8}$ of the pizza is left,” “Luis ate the most of 1 type of Pizza-cheese,” and, “Luis and Tito ate an equal amount of mushroom pizza.”
Representation Practitioner	The students diagram is appropriate to the problem and accurate. All pizzas are labeled, the proportions and labels are correct and a key defines Tito and Luis.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Practitioner/Expert, Student 1

Exemplars Rubric: Practitioner
 CCLS Content Rubric: Expert

This student is a Practitioner according to the Exemplars Rubric and an Expert according to the CCLS Content Standards Rubric (both included in the supporting materials).

Grade 5

Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

I have to find out who is correct
 Luis or Tito.
 I will make a chart.

boys	cheese Pizza eats	pepperoni Pizza eats	mushroom Pizza eats	Total	Converte
Tito	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{9}{8}$	$1\frac{1}{8}$
Luis	$\frac{5}{8}$	0	$\frac{1}{2}$	$\frac{9}{8}$	$1\frac{1}{8}$

Mathematical Work

Tito
 $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$
 $\frac{2}{8} + \frac{3}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$

Luis

$$\frac{5}{8} + \frac{1}{2}$$

$$\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$$

Answer Luis was correct

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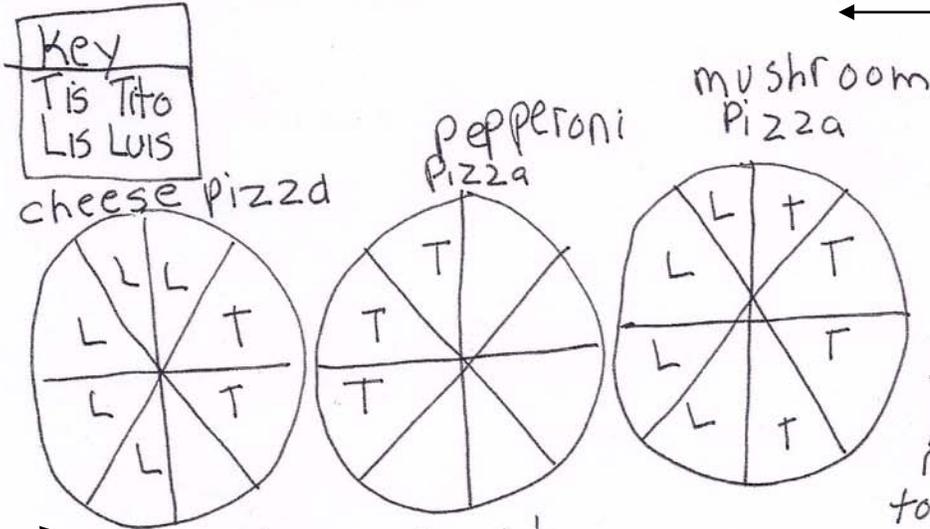
The student is able to make sense and persevere in solving the problem. The student demonstrates correct reasoning of proportional parts of a whole, correctly assigns each boy pizza pieces, and finds the correct equivalent fractions to state a correct answer. The student justifies her/his answer by using an area model/diagram as an alternative strategy.

The student uses precise mathematical terms-chart, diagram, key, most fractions, and fraction notation to help construct a viable argument.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Practitioner/Expert

The student shows evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student solves a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

I can prove by making a diagram of the slices.



L is 9 slices $\frac{9}{8} = 1\frac{1}{8}$
 T is 9 slices $\frac{9}{8} = 1\frac{1}{8}$

~~$\frac{9}{24} = \frac{3}{8}$~~
 no have to use eighths

I was correct The slices match my chart fractions

There are 24 slices if you also add the slices that didn't get eaten.
 Luis eats the most of one type of pizza - Cheese

The student models with mathematics. The chart is accurate with all columns labeled and all data accurate. The area model/diagram of the pizzas are accurate, labeled, and a key defines Tito and Luis. The student uses the area model/diagram to support that her/his answer is correct.

The student looks for and makes use of the structures in the problem. The student determines the boys eat twenty-four pieces and Luis eats the most of one type of pizza. The student evaluates the reasonableness of her/his results by using a diagram to verify that her/his answer is correct.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Practitioner/Expert

Practitioner/Expert – Student 1 Summary

Achievement Level: Student 1 is a Practitioner according to the Exemplars Process Rubric and an Expert according to the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Expert	The student’s strategy of using a chart to determine the fractional part of each pizza Tito and Luis eats, finding the total amount of slices eaten, and comparing the totals works to solve the problem. The student’s answer, “Luis was correct,” is correct. The student verifies her/his answer by using a diagram as an alternative strategy.
Reasoning and Proof Expert	The student demonstrates correct reasoning of proportional parts of a whole. The student correctly adds the eighths of pizza each boy eats for a total of $9/8 = 1\ 1/8$ and reasons correctly that Luis is correct. The student justifies her/his answer by using a diagram to represent the fractional amount of pizza each boy eats to verify that her/his answer was correct.
Communication Practitioner	The student correctly uses the terms-chart, diagram, key, most, fractions. The student correctly uses the mathematical notation- $1/4$, $3/8$, $1/2$, $9/8$, $1\ 1/8$, $5/8$, $2/8$, $4/8$.
Connections Expert	The student makes Practitioner connections. The student states, “There are 24 slices if you also add the slices that didn’t get eaten,” and, “Luis eats the most of one type of pizza-cheese.” The student makes an Expert connection. The student provides clarification and confirmation to the problem by solving the problem using two different strategies and then comparing them to determine if her/his answer is correct. The student states, “I was correct. The slices match my chart fractions.”
Representation Expert	The student’s chart is appropriate to the problem and accurate. The columns are labeled correctly and all data is accurate. The student’s diagram is appropriate and accurate. The proportions and all necessary labels are correct and a key defines Tito and Luis. The student uses the diagram to support that her/his chart was accurate resulting in a correct answer to the problem.

Instructional Implications: Stuffed with Pizza, 5

Student Achievement Level: Practitioner

The following is a list of instructional implications that you may want to consider for students performing at the Practitioner level. In addition, you may want to consult the suggestions for the Novice and Apprentice levels:

- Include more writing of equations using fractions.
- Encourage student to independently make more than one mathematically relevant connection about her/his work (see Preliminary Planning Sheet)
- Introduce another strategy to solve the same problem-area model/circle graph/diagram, table, tally chart, number line
- Substitute new fractions and determine a new total for Tito and Luis
- Investigate if changing the shape of the pizza will change the answer

Grade 5 Math: Stuffed with Pizza Annotated Student Work: Apprentice

Apprentice, Student 1
Name _____

This student is an Apprentice according to both the Exemplars Rubric and the CCLS Content Standards Rubric, (both included in the supporting materials).

The student is able to make sense of part of the problem. The student shows understanding of whole and eighths, although s/he omits one-eighth piece of cheese pizza for Tito. It appears that the student determines Luis is not correct because he ate two types of pizza and Tito ate all three types of pizza, which is an incorrect answer.

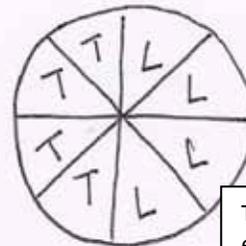
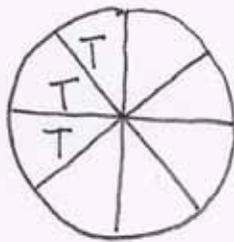
Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

The student uses the precise mathematical terms-diagram, most, more, to help construct her/his argument.

I have to find out who is correct.
I will make a diagram to show who ate the most pizza.

cheese pepperoni Mushroom



The student models with mathematics. All three pizzas demonstrate correct eighths but one-eighth of cheese pizza is missing a label for Tito.

Answer: Luis is not correct because he only ate the cheese pizza and mushroom pizza and Luis ate 9. Tito ate 8, 9 is more.

The student shows some evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student is not able to consistently solve a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer but omits one-eighth pizza for Tito. (5.NF 2).

The student is not able to search for structure, regularity, or trends embedded in the problem.

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Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Apprentice

Apprentice – Student 1 Summary

Achievement Level: Student 1 is an Apprentice according to both the Exemplars Process Rubric and the CCLS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student’s strategy of diagramming three pizzas to show the amount of pieces for each type of pizza Tito and Luis ate would work to solve the problem. The student omits a one-eighth piece of cheese pizza for Tito. The student’s answer, “Luis is not correct because he only did eat the cheese pizza and mushroom pizza and Luis ate 9, Tito ate 8. 9 is more,” is not correct.
Reasoning and Proof Apprentice	The student demonstrates correct reasoning that three whole pizzas are being considered in the problem. The student shows correct eighths in all three pizzas and assigns pieces of pizza to each boy. The student omits a one-eighth piece of cheese pizza for Tito which leads to an incorrect comparison of pieces eaten. It appears that the student may be misunderstanding the question. The student seems to be implying that Luis is not correct because he only ate cheese and mushroom pizza and Tito ate more “types” of pizza.
Communication Apprentice	The student correctly uses mathematical term “most,” from the problem. The student also correctly uses the term “diagram,” and more.
Connections Novice	The student solves the problem and stops without making a mathematically relevant observation about her/his solution.
Representation Apprentice	The student’s diagram is appropriate to the problem but is not accurate. A one-eighth piece of pizza for Tito is missing from the cheese pizza.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Apprentice

This student is an Apprentice according to both the Exemplars Rubric and the CCLS Content Standards Rubric (both included in the supporting materials).

Apprentice, Student 2
Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

The student is able to make sense of part of the problem. The student shows understanding of proportional parts of a whole. The student is unable to correctly calculate the total number of pieces each boy eats.

I have to find out how many pizza pieces both eat and who is correct.

Luis ate $\frac{1}{2}$ of the mushroom pizza and $\frac{5}{8}$ of the cheese pizza.

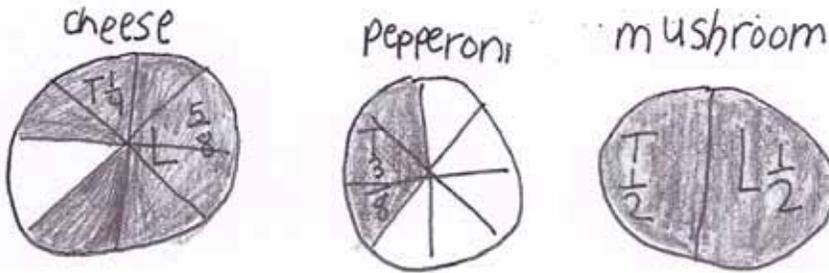
I will make a diagram and a key.

tito ate $\frac{1}{2}$ of the mushroom pizza and $\frac{3}{8}$ of pepperoni and $\frac{1}{4}$ of the cheese pizza!

page 1 of 2

Grade 5 Math: Stuffed with Pizza Annotated Student Work: Apprentice

The student models with mathematics. The area model/diagram of the pizzas are accurate, labeled and a key defines Tito and Luis.



The student shows little evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student is not able to consistently solve a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

The student uses precise mathematical terms-amount, diagram, key, and fraction notations to help construct a viable argument.

$$\frac{1}{2} + \frac{3}{8} + \frac{1}{4} = \frac{5}{20}$$

key
T tito
L Luis

$$\frac{5}{8} + \frac{1}{2} = \frac{6}{16}$$

Answer
Luis was
not correct

mushroom is the only pizza they eat a equal amount of.

The student is able to search for structure, regularity, or trends embedded in the problem. The student states, "mushroom is the only pizza they eat a equal amount of."

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Apprentice

Apprentice – Student Summary

Achievement Level: Student is an Apprentice according to both the Exemplars Process Rubric and the CCLS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student’s strategy of diagramming a cheese pizza, a pepperoni pizza, a mushroom pizza, shading in each piece eaten, and labeling each fractional part of the pizza eaten, would work to solve the problem. The student is not able to correctly find the total fractional part of the pizzas eaten by each boy to make a correct comparison. The student’s answer, “Luis was not correct,” is not correct.
Reasoning and Proof Apprentice	The student demonstrates correct reasoning of proportional parts of a whole. Each pizza is proportioned in eighths and the pieces each boy eats are labeled and shaded. The student is unable to correctly calculate the total number of pieces each boy eats. To find the total fraction for Tito the student appears to have multiplied the denominator two by the denominator eight and then adds four for a total denominator of twenty and adds the numerators one, three and one for a total numerator of five. For Luis the student appears to multiply the denominator eight by the denominator two for a total denominator of sixteen and adds the numerators five and one for a total numerator of six.
Communication Practitioner	The student correctly uses the mathematical term “amount,” from the problem. The student also correctly uses the terms “diagram” and “key.” The student correctly uses the mathematical notation- $\frac{1}{4}$, $\frac{5}{8}$, $\frac{3}{8}$, $\frac{1}{2}$. The student does not earn credit for the fractions- $\frac{5}{20}$, $\frac{6}{16}$, because they are not accurate for this problem.
Connections Practitioner	The student makes the mathematically relevant observation, “mushroom is the only pizza they eat a equal amount of.”
Representation Practitioner	The students diagram is appropriate to the problem and accurate. All pizzas are labeled, the proportions and fraction labels are correct and a key defines Tito and Luis.

Grade 5 Math: Stuffed with Pizza

Annotated Student Work: Apprentice/Practitioner, Student 1

Exemplars Rubric: Apprentice
CCLS Content Rubric: Practitioner

This student is an Apprentice according to the Exemplars Rubric and a Practitioner according to the CCLS Content Standards Rubric (both included in the supporting materials).

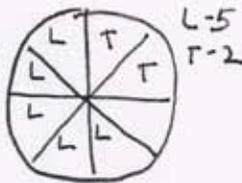
Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

I have to find out who is correct Luis or Tito?

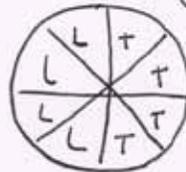
I will draw a diagram to figure out who ate the most pizza



L-5
T-3



L-0
T-3



L-4
T-4

key
T tito
L Luis

cheese pizza
my work -
Luis ate $\frac{5}{8} + \frac{1}{2}$

$$\frac{5}{8} + \frac{4}{8} = \frac{9}{8} = 1\frac{1}{8}$$

Tito ate 9 slices

$$\frac{1}{4} + \frac{1}{2} + \frac{4}{8} = \frac{2}{8} + \frac{4}{8} + \frac{4}{8} = \frac{10}{8} = 1\frac{2}{8}$$

Answer
Luis is correct

The student is able to make sense and persevere in solving the problem. The student demonstrates correct reasoning of proportional parts of a whole, correctly assigns each boy pizza pieces, and finds the correct equivalent fractions to state a correct answer.

The student uses precise mathematical terms: most, diagram, key, and fraction notation to help construct a viable argument.

The student models with mathematics. The area model/diagram of the pizzas are appropriate, labeled, and a key defines Tito and Luis.

The student shows evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student solves a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

The student does not reflect on her/his solution to make connections about the structures, regularities, or trends in the problem.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Apprentice/ Practitioner

Apprentice/Practitioner - Student 1 Summary

Achievement Level: Student 1 is an Apprentice according to the Exemplars Process Rubric and a Practitioner according to the CCLS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student’s strategy of diagramming a cheese pizza, a pepperoni pizza, a mushroom pizza, labeling each fractional part of the pizza eaten, finding the total amount each boy ate and comparing those totals, works to solve the problem. The student’s answer, “Luis is correct,” is correct.
Reasoning and Proof Practitioner	The student demonstrates correct reasoning of proportional parts of a whole. Each pizza is proportioned in eighths and the pieces each boy eats are labeled. The student correctly finds the correct equivalent fractions in eighths for one-half, and one-fourth. The student correctly adds the fractions and compares the sum to determine that Luis was correct.
Communication Practitioner	The student correctly uses the mathematical terms “most,” “diagram,” and “key.” The student correctly uses the mathematical notation $\frac{5}{8}$, $\frac{1}{2}$, $\frac{4}{8}$, $\frac{9}{8}$, $1\frac{1}{8}$, $\frac{1}{4}$, $\frac{3}{8}$,
Connections Apprentice	The student solves the problem and does not make mathematically relevant observations about her/his solution.
Representation Practitioner	The students diagram is appropriate to the problem and accurate. All pizzas are labeled, the proportions and labels are correct and a key defines Tito and Luis.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work

Instructional Implications: Stuffed with Pizza, 5

Student Achievement Levels: Novice and Apprentice

The following is a list of instructional implications that you may want to consider for students performing at the Novice and Apprentice levels:

- Review how to read a mathematics problem-listen to the problem being read, read the problem to yourself, underline the important information, find the question sentence(s), determine the important nouns, look for mathematical language, etc.
- Use manipulatives to investigate fractional part of a whole-fraction bars and circles, fraction wheels, paper plates, sticks
- Use games requiring the use of fractions with and without like denominators
- Work with a number line
- Order fractions with cards, sticks, etc.
- Provide activities where student adds and subtracts fractions with like and unlike denominators
- Review mathematical language-model, diagram/area model/circle graph key, table, number line, more/less than, fraction, $\frac{1}{2}$, $\frac{3}{4}$, $\frac{2}{8}$..., numerator, denominator, whole, equivalent
- Review how to make a diagram/area model with a key, table, chart, number line
- Have centers available for investigation and practice with finding and applying fractions
- Provide leading questions to begin reflection on the solution in order to see regularities, structures, patterns, trends, etc. (See Preliminary Planning Sheet)

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Novice/Apprentice, Student 1

Exemplars Rubric: Novice
CCLS Content Rubric: Apprentice

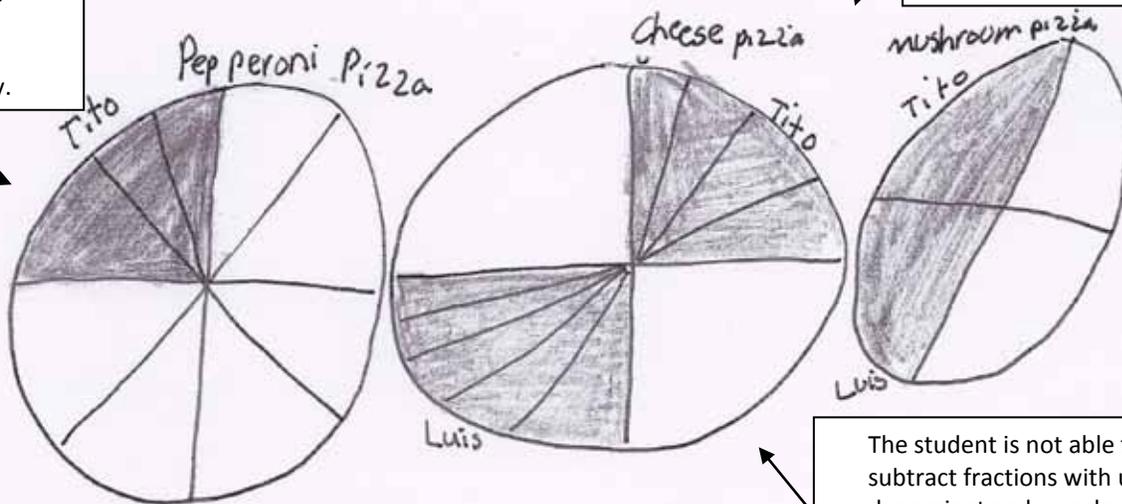
This student is a Novice according to the Exemplars Rubric and an Apprentice according to the CCLS Content Standards Rubric (both included in the supporting materials).

Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

I think Tito ate a lot then Luis



*answer
 I was wrong
 Tito ate 8 and
 Luis ate 6 pieces*

The student attempts to model with mathematics but the area model/diagram is not equally proportioned or shaded correctly.

The student is not able to discern that the problem requires an understanding of fractional parts to a whole and comparing parts of the whole each boy eats. The student's answer is not correct.

The student is not able to add and subtract fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student is not able to solve a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

The student is not able to search for the regularity and trends embedded in the problem.

The student does not use any precise mathematical terms to support her/his argument.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Novice/Apprentice

Novice/Apprentice – Student 1 Summary

Achievement Level: Student 1 is a Novice according to the Exemplars Process Rubric and an Apprentice according to the CCLS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student’s strategy of diagramming three pizzas does not work to solve the problem because the student is not able to determine the correct proportional parts for each pizza. The student’s answer, “I was wrong. Tito ate 8 and Luis ate 6 pieces,” is not correct.
Reasoning and Proof Apprentice	The student does demonstrate correct reasoning that three whole pizzas are being considered in the problem. The student shows no other understanding of the other underlying mathematical concepts in the problem. The student does not apply correct reasoning of fractional parts of a whole when considering fourths, eighths, and halves. The student is not able to count equal slices of pizzas for Tito and Luis or compare fractions with unlike denominators.
Communication Novice	The student does not use any mathematical language.
Connections Novice	The student solves the problem and stops without making a mathematically relevant connection.
Representation Apprentice	The student attempts to make a diagram. The student diagrams the pepperoni, cheese, and mushroom pizzas but is not able to show fourths, eighths or halves. It appears that the mushroom pizza has one-half shaded but the student is indicating one-fourth as the half a mushroom pizza that Tito eats and one-fourth as the half a mushrooms pizza that Luis eats.

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Novice/Apprentice, Student 2

Exemplars Rubric: Novice
CCLS Content Rubric: Apprentice

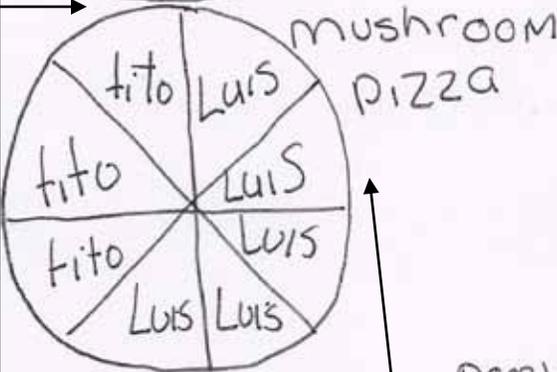
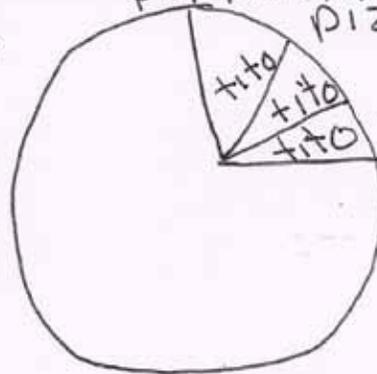
This student is a Novice according to the Exemplars Rubric and an Apprentice according to the CCLS Content Standards Rubric (both included in the supporting materials).

Name _____

Stuffed with Pizza

Tito and Luis are stuffed with pizza! Tito ate one-fourth of a cheese pizza. Tito ate three-eighths of a pepperoni pizza. Tito ate one-half of a mushroom pizza. Luis ate five-eighths of a cheese pizza. Luis ate the other half of the mushroom pizza. All the pizzas were the same size. Tito says he ate more pizza than Luis because Luis did not eat any pepperoni pizza. Luis says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

I have to find out who ate the biggest pizza pieces. I will do a diagram.



tito and Luis ate the same and Luis was correct

The student uses the precise mathematical term-diagram, to support her/his argument.

The student is able to make sense of part of the problem. The student understands three whole pizzas, and correct eighths in the cheese pizza as well as the correct amount Tito and Luis eat. The pepperoni pizza is proportioned incorrectly and the mushroom pizza assigns an extra eighth of pizza to Luis and one less to Tito. The student's answer is not correct based on her/his reasoning.

The student attempts to model with mathematics. The cheese pizza area model/diagram is accurate. The amount each boy eats in the mushroom pizza area model/diagram is not accurate. The pepperoni pizza area model/diagram does not show correct understanding of one-fourth of a pizza being equal to two-eighths of the pizza.

The student is not able to search for structure, regularity or trends embedded in the problem.

The student shows some evidence in adding fractions with unlike denominators by replacing given fractions with equivalent fractions. (5.NF 1). The student is not able to consistently solve a word problem involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators by using fraction models or equations to represent the problem, or use benchmark fractions and number sense to estimate mentally and assess the reasonableness of the answer (5.NF 2).

Grade 5 Math: Stuffed with Pizza
Annotated Student Work: Novice/Apprentice

Novice/Apprentice – Student 2 Summary

Achievement Level: Student 2 is a Novice according to the Exemplars Process Rubric and an Apprentice according to the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student’s performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student’s strategy of diagramming three pizzas to show the amount of pieces for each type of pizza Tito and Luis ate would work to solve the problem but the student is not able to show correct proportional parts for all pizzas or slices eaten by Tito and Luis. The student is not able to correctly compare the number of slices s/he assigned each boy. The student’s answer, “tito and Luis ate the same and Luis was correct,” is not supported by her/his solution and is therefore not credited.
Reasoning and Proof Apprentice	The student demonstrates correct reasoning that three whole pizzas are being considered in the problem. The student shows some understanding of the other underlying mathematical concepts in the problem. The student shows correct eighths in the cheese pizza and assigns Tito and Luis the correct amount of pieces eaten. The student does not show correct eighths in the pepperoni pizza diagram. It appears the student made a fourth and proportioned that fourth into three-eighths pizza eaten by Tito. The student shows correct eighths in the mushroom pizza but assigns an extra eighth to Luis and one less to Tito showing incorrect reasoning of one-half. The student states, “Tito and Luis ate the same,” but there are eight slices of pizza assigned to Tito and ten slices of pizza assigned to Luis.
Communication Apprentice	The student correctly uses the mathematical term “diagram.”
Connections Novice	The student solves the problem and stops without making a mathematically relevant connection.
Representation Apprentice	The student attempts to make a diagram. The student correctly diagrams the cheese pizza and correctly labels the slices that Tito and Luis eat. The pepperoni pizza is not proportionally correct. The mushroom pizza is proportionally correct but the slices for both Tito and Luis are incorrect.



GRADE 5 MATH: STUFFED WITH PIZZA

INSTRUCTIONAL SUPPORTS

The instructional supports on the following pages include a unit outline with formative assessments and suggested learning activities. Teachers may use this unit outline as it is described, integrate parts of it into a currently existing curriculum unit, or use it as a model or checklist for a currently existing unit on a different topic.

Unit Outline –Grade 5 Math

INTRODUCTION: In this unit, students will develop and expand the concept of rational numbers by using several interpretations and different types of physical models. Students will explore their own ways of inventing algorithms to perform operations with rational numbers and they will also understand the mathematics behind the traditional algorithms to do operations with rational numbers. The outline provides an example of how teachers may integrate performance tasks into a unit. *Teachers may (a) use this unit outline as it is described below; (b) integrate parts of it into a currently existing curriculum unit; or (c) use it as a model or checklist for a currently existing unit on a different topic.*

Grade 5 Math: Understanding Rational Numbers

UNIT TOPIC AND LENGTH:

- Ø Making sense of the meaning of fractions, decimals and percents in different contexts
- Ø 40-50 days

COMMON CORE LEARNING STANDARDS:

Number & Operations—Fractions 5.NF

Use equivalent fractions as a strategy to add and subtract fractions.

- Ø **5.NF.1.** Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. *For example, $2/3 + 5/4 = 8/12 + 15/12 = 23/12$. (In general, $a/b + c/d = (ad + bc)/bd$.)*
- Ø **5.NF.2.** Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. *For example, recognize an incorrect result $2/5 + 1/2 = 3/7$, by observing that $3/7 < 1/2$.*

Apply and extend previous understandings of multiplication and division to multiply and divide fractions.

- Ø **5.NF.3.** Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. *For example, interpret $3/4$ as the result of dividing 3 by 4, noting that $3/4$ multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size $3/4$. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*
- Ø **5.NF.4.** Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
 - a) Interpret the product $(a/b) \times q$ as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations $a \times q \div b$. *For example, use a visual fraction model to*

Unit Outline –Grade 5 Math

show $(2/3) \times 4 = 8/3$, and create a story context for this equation. Do the same with $(2/3) \times (4/5) = 8/15$. (In general, $(a/b) \times (c/d) = ac/bd$.)

- b) Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths.

Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.

- Ø **5.NF.5.** Interpret multiplication as scaling (resizing), by:
- Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
 - Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1.
- Ø **5.NF.6.** Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.
- Ø **5.NF.7.** Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions.
- Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. *For example, create a story context for $(1/3) \div 4$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $(1/3) \div 4 = 1/12$ because $(1/12) \times 4 = 1/3$.*
 - Interpret division of a whole number by a unit fraction, and compute such quotients. *For example, create a story context for $4 \div (1/5)$, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that $4 \div (1/5) = 20$ because $20 \times (1/5) = 4$.*
 - Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. *For example, how much chocolate will each person get if 3 people share $1/2$ lb of chocolate equally? How many $1/3$ -cup servings are in 2 cups of raisins?*

Number & Operations in Base Ten 5.NBT

Understand the place value system.

- Ø **5.NBT.1.** Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and $1/10$ of what it represents in the place to its left.
- Ø **5.NBT.2.** Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole number exponents to denote powers of 10.
- Ø **5.NBT.3.** Read, write, and compare decimals to thousandths.
- Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g.,
 - $347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times (1/10) + 9 \times (1/100) + 2 \times (1/1000)$.
 - Compare two decimals to thousandths based on meanings of the digits in each place, using $>$, $=$, and $<$
 - symbols to record the results of comparisons.

Unit Outline –Grade 5 Math

<p>∅ 5.NBT.4. Use place value understanding to round decimals to any place.</p> <p>Perform operations with multi-digit whole numbers and with decimals to hundredths.</p> <p>∅ 5.NBT.7. Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used</p>	
<p>BIG IDEAS/ENDURING UNDERSTANDINGS:</p> <ul style="list-style-type: none"> ∅ Fractional parts are like equal shares of a whole thing or a whole set, which depends on the part of the whole, and equivalent fractions describe the same amount using different fractional parts. ∅ The numerator of a fraction tells how many parts of the whole are considered, and the denominator indicates the whole. ∅ Exploring, understanding, and connecting real life situations where fractions, decimals, and percent concepts are essential. 	<p>ESSENTIAL QUESTIONS:</p> <ul style="list-style-type: none"> ∅ How can I describe fractions and equivalent fractions? ∅ What do the parts of fractions mean? ∅ In everyday life, how can I convert between fractions, decimals, and percents?
<p>CONTENT:</p> <p>Interpretations of fractions</p> <ul style="list-style-type: none"> ∅ Parts of a whole ∅ Part of a set ∅ Measures or quantities ∅ Equivalents <p>Parts of a fraction</p> <ul style="list-style-type: none"> ∅ Numerator ∅ Denominator <p>Connections of fractions</p> <ul style="list-style-type: none"> ∅ Operations to convert between fractions, decimals and percents ∅ Context clues ∅ Strategies to make sense of problems ∅ Algorithms with fractions, decimals and percents ∅ Relationships between fractions, decimals, and percents 	<p>SKILLS:</p> <p>Interpretations of fractions</p> <ul style="list-style-type: none"> ∅ Construct and apply concepts of equivalent fractions to reason about situations ∅ Connect concepts of fractions with the symbolic representation ∅ Construct pictorial models for fractions (for example, showing $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$ by drawing an area model) <p>Parts of a fraction</p> <ul style="list-style-type: none"> ∅ Communicate and identify appropriate mathematical terminologies to refer to fractional parts <p>Connections of fractions</p> <ul style="list-style-type: none"> ∅ Develop strategies for adding and subtracting fractions and decimals. ∅ Explore different models to understand the concepts of fractions, decimals, and percents ∅ Develop ways to model situations involving fractions, decimals, and

Unit Outline –Grade 5 Math

percents

- Ø **Convert** between benchmarks that relate different forms of representations of rational numbers (for example 50% is the same as $\frac{1}{2}$ and .5)
- Ø **Show** physical models and drawings to help reason about a situation
- Ø **Identify** when addition or subtraction of fractions is the appropriate operation
- Ø **Identify** context clues to help solve a problem

KEY TERMS/ VOCABULARY:

Decimal, denominator, equivalent fractions, fractions, numerator, percent, algorithm, benchmarks, mixed numbers, diagrams, model, fraction bar, fraction strips, base ten number system, patterns, sums, products, differences, estimation, factors, grid, number lines, more than, less than, quotient, total

ASSESSMENT EVIDENCE AND ACTIVITIES:

INITIAL ASSESSMENT : AUTUMN FUN

This activity will be given to students at the beginning of the unit. This task will give teachers an insight into students' level of conceptual understanding about fractions, decimals, and percents and the relationships among these concepts. Teachers will be able to assess the types of representations students are using to solve the problem. In addition, teachers will be able to diagnose how students understand the concept of rational numbers in a real life situation context. *See Autumn Fun for details.*

FORMATIVE ASSESSMENT: EATING PIE

This activity gives an opportunity to students to use appropriate physical models to work with fractions in real life context. In addition, students will be able to compare different physical models such as circles and fraction strips and decide which ones are more efficient dependent in the context of the problem.

This task provides teachers with insights about students' knowledge in comparing bench-mark fractions with unlike denominators by using physical models and relating them to the symbolic representations. The task also gives teachers insights about the foundational concepts students have acquired with equivalent fractions. *See Eating Pie for details.*

Please Note: There are other tasks that have been included in the support materials (referenced in the learning plan section) that can be used to deepen students' understanding of fractions as you move through the unit after investigation 2.

FINAL PERFORMANCE TASK : STUFFED WITH PIZZA

This task assess students' understanding about the part-whole interpretation of fractions , how they use

Unit Outline –Grade 5 Math

different physical models to represent and compare bench mark fractions, and how the physical models relate to the symbolic representations of fractions. *See Stuffed With Pizza for details.*

LEARNING PLAN & ACTIVITIES:

Please see the Resources section for the information on the books referenced below.

Lessons and Activities

1. Introduce the initial activity-“**Autumn Fun**” by providing students with different types of manipulatives (Connecting Blocks, Color Tiles, Fractional Bars or Fractional Circles, Quissionaire Fraction Bars, Color markers or crayons, grid or dot papers, number lines, calculators, Geoboards, Pattern Blocks,)
2. **Investigation 1.1**-“Fun Raising Fractions”: Assessing Students’ Understanding of Fractions as Parts of Wholes”, Connected Math-- Bits and Pieces I, Student Book page 5, Teacher’s Guide 18a-18b
(1 day)
3. **Investigation 1.2**-“ Using Fraction Strips”: To Relate Fraction Strips Models to the Part-Whole Interpretation of Fractions and the Symbolic Representation of Fractions”, Connected Math-- Bits and Pieces I, Student Book pages 6-7, Teacher’s Guide 18b-18f
(1 day)
4. **Investigation 1.3**-“Comparing Classes”: Comparing Fractions With Different Wholes, “ Connected Math-- Bits and Pieces I, Student Book pages 8-9, Teacher’s Guide 18f-18g
(1 day)
5. **Investigation 1.4** –“Exceeding the Goal“ To Understand The meaning of Fractions Larger Than the Whole”: , Connected Math-- Bits and Pieces I, Student Book pages 10-11, Teacher’s Guide 18g-18i
(1 day)
6. **Investigation 1.5** -“Using Symbolic Form ”: To Use Fraction Strips and Symbolic Representation of Fractions To describe Real Word Situations” , Connected Math-- Bits and Pieces I, Student Book pages 12-13, Teacher’s Guide 18i-18j
(1 day)
7. Introduce Formative Activity: “**Eating Pie**” by providing students with different types of manipulatives (Connecting Blocks, Color Tiles, Fractional Bars or Fractional Circles, “Quissionaire Rods” , Color markers or crayons, grid or dot papers, number lines, calculators, Geo-boards, Pattern Blocks,) (1 day)
8. **Investigation 2**- Connected Math-- Bits and Pieces I, -Connecting Fraction Strips and Number Lines to Build on the Concept of Fractions, Student Book pages 19-30, Teacher’s Guide 30a-30k
Mathematical Goals:
 - To continue to use fraction strips as tools for understanding fraction concepts
 - To investigate the concepts of comparison and equivalence of fractions

Unit Outline –Grade 5 Math

- To use fractions that are equal to , and greater than 1
- To apply knowledge gained by using fraction strips name, estimate , and compare fractions and to find equivalent fractions
- To build on number line and labels points between whole numbers

(5 days)

9. Assess students with the tasks from the Summative Activities listed below while teaching the lessons from **Investigation 2.**

- “Favorite Sports”
- “New Books for the Library”
- “A Field Trip”

10. **Investigation 3- Connected Math-- Bits and Pieces I.** -Modeling Fractions as Subdivisions of Areas of Figures, Student Book pages 31-38, Teacher’s Guide 38a-38g

Mathematical Goals:

- To continue building an understanding of equivalent fractions
- To explore the use of squares and other areas as a way to build visual models of fractional parts of a whole- **1 day**
- To explore real -life problems that require operations on fractions in a context that invites the use of informal strategies rather than formal rules and algorithms

(3 days)

11. Assess students with the tasks from the Summative Activities listed below while teaching the lessons from **Investigation 3.**

- “Brownies”
- “Lots of Cakes”

12. **Investigation 4- Connected Math-- Bits and Pieces I.** -Using Square Grids as Contexts To Introduce Decimal Numbers and Relating Them To Other Models With Fractions; Student Book pages 39-352, Teacher’s Guide 52a-52k

Mathematical Goals:

- Representing Fractional Parts of a Whole Using Square Grids
- Understanding Decimal Place Values by Representing Them Graphically (up to the ten thousandths)
- Using Decimal Bench-marks and Relating Them To Fractions (**1day**)
- Furthering Understanding of Place Value by Playing the game “ The Distinguished Digits”

(4 days)

13. Assess students with the tasks from the Summative Activities listed below while teaching the lessons from **Investigation 4.**

- “Coins in Grandpa’s Pockets”
- “New Balls for Recess”
- “Pizza For a Party”
- “Eating Pie”

14. **Investigation 5- Connected Math-- Bits and Pieces I.** -“Developing Meaning of Fractions as Implied Division” Student Book pages 53-66, Teacher’s Guide 66a-66k

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Mathematical Goals:

- To Make Comparison Among Three Quantities Using Fractions
- To Write Fractions As Decimals, Represent Fractions Using Hundredth Strips, Fraction Strips, and To Estimate Fractions and Decimal Equivalents,
- To Understand Why Fractions Can Be Interpreted As Implied Divisions and To Use Implied Division to Change Fractions to Decimals Representations

(3 days)

15. Assess students with the tasks from the Summative Activities listed below while teaching the lessons from **Investigation 5**.

- “Cherry Pies”

16. **Investigation 6- Connected Math-- Bits and Pieces I** -Using Two Ways To Visualize Percents, Student Book pages 67-83, Teacher’s Guide 83a-84

Mathematical Goals:

- To Use “Out of 100” Interpretation of Fractions and Decimals To Develop an Understanding of Percents
- To Use The Hundredths Grid To Visualize the Concept of Percents as Meaning “Out of 100”
- To Investigate the Relationships Among Fractions , Decimals, and Percents and To Move Flexibility Among Representations
- To apply knowledge gained by using fraction strips name, estimate , and compare fractions and to find equivalent fractions
- To Understand How To Use Percent as An Expression of Frequency, In terms of “Out of 100,” When a Set of Data has More or Fewer Than 100 Items

(4 days)

17. **Investigation 3- Connected Math-- Bits and Pieces II** -Computations With Fractions and Decimals Using Estimations, Student Book pages 31-42, Teacher’s Guide 42a-42c

Mathematical Goals:

- To Develop Strategies For Estimating Sums of Fractions and Decimals
 - To Make Sense of Whether a Situation Requires an Overestimate or an Underestimate
- 1day**
- To use 0, $\frac{1}{2}$, 1, $1\frac{1}{2}$, and 2 as Benchmarks
 - To make Sense of The Size of A Sum
 - To Use Estimation Strategies to Quickly Approximate a Particular Sum,
 - To build on number line and labels points between whole numbers

(3 days)

18. **Investigation 4- Connected Math-- Bits and Pieces II** -Making Sense of Algorithms For Adding and Subtracting Fractions, Student Book pages 45-53, Teacher’s Guide 53a-53i

Mathematical Goals:

- To Use The Area Model For Adding and Subtracting Fractions
- To Raise the Need To Multiply, Divide, Add and Subtract Fractions; And To Model Ways To Do Computation With Fractions
- To Design and Reflect in Algorithms and Strategies For Adding and Subtracting Fractions
- To apply knowledge gained by using fraction strips name, estimate , and compare fractions and to find equivalent fractions
- To build on number line and labels points between whole numbers

(5 days)

Unit Outline –Grade 5 Math

19. **Investigation 5- Connected Math-- Bits and Pieces II** -Using The Area Model To Represent Fractions of a Fraction, Student Book pages 54-63, Teacher’s Guide 63a-63i

Mathematical Goals:

- To find fractional parts of a fraction using the area model
- To work with mixed numbers using the area model
- To multiply fractions-using the area model
- To construct and design a multiplication algorithms
- To build on number line and labels points between whole numbers

(5 days)

20. Developing Understanding To the Division Algorithm With Fractions, Elementary and Middle School Mathematics, Teaching Developmentally, Third Edition; John A. Van de Walle, pages 269-273 (2days)

21. **Investigation 7- Connected Math-- Bits and Pieces II** -Dividing Fractions, Student Book pages 7987-63, Teacher’s Guide 162a-164

Mathematical Goals:

- To Use Contexts To Understand the Concept of Dividing Fractions–A Whole Number Divided By a Fraction
- To Use Patterns That Can Lead To an Efficient Algorithm to Divide Fractions (3 days)

22. **Investigation 6- Connected Math-- Bits and Pieces II** - Adding, Subtracting , Multiplying and Dividing Decimals, Student Book pages 64-76, Teacher’s Guide 76a-76j

Mathematical Goals:

- To Use the Decimal Notation for Money To Help Them Practice Estimating Sums and Differences of Decimal Numbers
- To Analyze the Movement of Decimal Point and The Effects of the Placement of the Decimal Point on Sums and Differences
- Searching for Patterns To Multiply Decimals-What happen to the Decimal Point?
- Shifting The Decimal Points in Factors to Obtain a Given Product or a Product in a Given Range
- Solving Problems In a Real-world Contexts That Involve Computations With Fractions and Decimals (5 days)

Evidence of Students’ Learning

1. **Journal Writing**

- a. Students may be directed to reflect on the answers to essential questions at the end of each investigation: Mathematical Reflection Pages from Bits and Pieces I and II

2. **Students may be exposed to different types of activities as listed below during the mathematics blocks , as extensions for homework, tests and quizzes**

- a. Answers individual problems for each investigation
- b. Unit Reflections
- c. Assessment Resources from the Units: Check Ups, Quizzes, Unit Tests, Individual Research, Self Assessments, Additional Practice Problems

Unit Outline –Grade 5 Math

Extension Activities- *use as needed to build and reinforce concepts and challenge students*

1. Everyday Mathematics, Grade 5, Chapters 2, 5 and 8
2. Fractions, Decimals, Ratios, and Percents, Hard To Teach and Hard To Learn; Carne Barnett, Donna Goldenstein, Babette Jackson

RESOURCES: RESOURCES:

Texts

- Ø 1. Elementary and Middle School Mathematics, John A. Van de Walle, Third Edition; chapters 12-14 pages 237-290
- Ø Connected Mathematics-Number and Operations, Prentice Hall, Bits and Pieces I and II, Understanding Rational Numbers and Using Rational Numbers
- Ø Everyday Mathematics, Grade 5, Chapter 2 (Estimation and Computation) pages 66-138; Chapter 5, pages 276-30, Chapter 8, pages 604-686
- Ø Fractions, Decimals, Ratios, and Percents, Hard To Teach and Hard To Learn; Carne Barnett, Donna Goldenstein, Babette Jackson

Interdisciplinary Connections

- Ø **Art-**Students can design a floor plan of a house and represent the different parts of the design using multiple representations with rational numbers.
- Ø **Language Arts-** Students can write an essay responding to the following prompt: Can we survive without the existence of rational numbers? Explain.

P/S	R/P	Com	Con	Rep	AC/LV

Name _____

Autumn Fun

The Outdoors Club members are planning some activities to do on a sunny autumn Saturday. One half of the members are going hiking. One fourth of the members are bike riding. One eighth of the members are rock climbing. Nine of the members are walking their dogs. How many Outdoor Club members are doing an activity on Saturday? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task ⁽⁵⁾

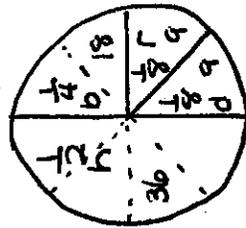
Title of Task Autumn Fun Content Strand(s) Addressed Number+operations fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1,2

Underlying Mathematical Concepts

fractional part of a whole
 number sense to 72
 multiplication (addition/division)
 comparing fractions with unlike denominators

Answer
 72 members

Possible Solution(s)
 members/activities



Key
 h hiking
 b bike riding
 r rock climbing
 d dog walking
 m members

$$h + b + r + d = m$$

$$36 + 18 + 9 + 9 = 72$$

Activity	fraction of members	percent of members	decimals of members
hiking	$\frac{1}{4}$	50%	.50
biking	$\frac{1}{8}$	25%	.25
rock climb.	$\frac{1}{8}$	12.5%	.125
dog walk.	$\frac{1}{8}$	12.5%	.125

Problem Solving Strategies/Representation
 area model (circle graph) | bar graph
 table / chart
 model - fraction bars | circles

Mathematical Language
 area model
 arc | graph
 model
 table
 chart
 key
 fraction's $\frac{1}{4}$...
 decimals .25 ...
 percents 25% ... ratio
 numerator
 denominator
 most/least likely

Connections

- hiking is the mode
- find the decimal part / percent part for each activity
- a new student most likely would select hiking
- Relate to a similar problem and state math
- $\frac{1}{8} < \frac{1}{4}$ so more students prefer biking to rock climbing or dog walking
- verify by solving with a new strategy
- hiking is the maximum dog walking and rock climbing are equivalent

Related Tasks
 See Resource Binder

ratio $\frac{36}{72}$ select hiking

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Eating Pie

Dad bakes a cherry pie, an apple pie, and a blueberry pie. All the pies are the same size. Alexis eats one-sixth of the cherry pie, one-half of the apple pie, and three-sixths of the blueberry pie. Francisco eats four-sixths of the cherry pie, three-sixths of the apple pie, and one-third of the blueberry pie. Who eats the most pie? How much of each pie is left over? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Eating Pie Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link Everyday Mathematics Unit 5
 Common Core Standard(s) _____

Underlying Mathematical Concepts fractional parts of a whole Problem Solving Strategies/Representation model (manipulatives/fraction circles)
improper / proper fractions diagram / key - area model circle graph
mixed fractions chart
comparison of decimals
with unlike denominators

Mathematical Language
 model whole
 area model equivalent
 circle graph per
 diagram greater/less
 key than
 more/less than $>, <$
 fractions $\frac{2}{6}, \frac{1}{3}, \dots$
 percent
 decimal
 numerator
 denominator

Possible Solution(s)

Pies
 cherry apple

$A - \frac{2}{6} = 1\frac{1}{6}$
 $F - \frac{3}{6} = 1\frac{3}{6} = 1\frac{1}{2}$

	Pies Eaten		Pies not Eaten
	cherry	apple	
A Alexis	$\frac{2}{6}$	$\frac{4}{6} = 1$	$\frac{1}{6}$
F Francisco	$\frac{3}{6}$	$\frac{3}{6}$	$\frac{1}{6}$

Answer
 Francisco $\frac{1}{6}$ cherry
 Alexis $\frac{0}{6}$ apple
 Alexis $\frac{1}{6}$ blueberry
 Alexis $\frac{0}{6}$ blueberry

- Connections
- Alexis eats $1\frac{1}{6}$ pie, but not a whole pie.
 - both ate $\frac{1}{2}$ of 50% or .5 of the apple pie
 - Alexis ate equal amounts of apple and blueberry pie
 - Ate a total of $1\frac{1}{6}$ or $2\frac{4}{6}$ or $2\frac{2}{3}$ pie
 - Alexis ate $33\frac{1}{3}\%$ of cherry pie or .333
 - $\frac{2}{6}$ or $\frac{1}{3}$ slices are left
 - Relate to a similar problem and state math.

Related Tasks
 See Resource Binder

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Pizza for a Party

The fifth graders are planning a pizza party. Each fifth grader will eat two slices of pizza with a favorite topping. Each fifth grader selects a favorite topping. One-half of the students want pepperoni. One-fourth of the students want extra cheese. One-eighth of the students want sausage. Four students want mushrooms. How many students select a favorite topping for their pizza slices?

The local pizza shop sells a large whole pizza with one topping and cut in eight equal pieces for nine dollars and ninety-six cents. But, if you buy one pizza at the regular price you can get the second large pizza with the same topping for fifty percent off. How many different large pizzas will be needed for the pizza party and what will be the total cost?

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task 5

Title of Task Pizza for a Party Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts
 fractional part of a whole
 money notation
 addition / multiplication
 2 to 1 (2 slices to 1 student)
 comparing fractions with unlike denominators

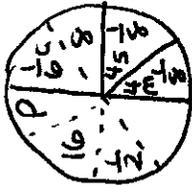
Problem Solving
 Strategies/Representation
 model (manipulatives - fraction bars/circles)
 circle graph/bar graph/area model
 table
 chart

Mathematical Language
 model
 decimals .50
 circle graph
 table
 chart
 fractions $\frac{1}{2}$... ratio
 numerator
 denominator
 data
 more/less likely
 mode
 minimum
 maximum
 per

Related Tasks
 See Resource Binder

Answer
 32 students
 4 pepperoni
 2 cheese
 1 mushroom
 1 sausage
 \$ 67.14

Key
 P pepperoni
 C extra cheese
 S sausage
 M mushroom
 T Total students



Possible Solution(s)

Students + Pizza Toppings

$p + c + s + m = T$
 $16 + 8 + 4 + 4 = 32$

Toppings	students	# slices	# pizzas	cost
P	16	32	4	\$29.88
C	8	16	2	\$14.94
M	4	8	1	\$9.96
S	4	8	1	\$9.96

Connections
 • Pepperoni is the mode
 • Find percent for each topping

Find decimal for each topping

no extra slices

64 total slices

You save \$14.94

Relate to a similar problem and state math

A new student most likely will ask for pepperoni topping
 • sausage and mushroom was equivalent

even number of students in each category.
 • ratio $\frac{16}{32}$ select pepperoni

Name _____

P/S	R/P	Com	Con	Rep	Ach/Level

New Balls for Recess

Mr. Mitchell needs to order new balls for the Lincoln School students to use at recess. He asks the students what balls he should order. One-third of the students want Mr. Mitchell to order soccer balls. One-third of the students want Mr. Mitchell to order basketballs. One-sixth of the students want Mr. Mitchell to order footballs. Fifteen students want Mr. Mitchell to order kickballs. How many students did Mr. Mitchell ask about ordering new balls for recess?

Mr. Mitchell decides to buy all the balls his students want him to order. He wants to order eight soccer balls that cost \$8.29 per ball and eight basketballs that cost \$9.49 per ball. He wants to order four footballs that cost \$10.50 per ball and four kickballs that cost \$6.95 per ball. Mr. Mitchell has a budget of \$225.00 for ordering balls. Will Mr. Mitchell be able to buy all the balls he wants to order? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task New Balls for Recess Content Strand(s) Addressed Number + Operations Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts

fractional part of a whole
 money notation
 number sense to \$212.04
 multiplication / addition
 comparing fractions with unlike denominators
 comparing totals

Answers
 90 students
 Yes he has enough money

Possible Solution(s)

Students + Balls

30	30	15	15
$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{6}$
S	b	f	K

$S + b + f + K = 90$
 $30 + 30 + 15 + 15 = 90$

balls ordered	ball type	cost per ball	total cost
8	S	\$8.29	\$66.32
8	b	\$9.49	\$75.92
4	f	\$10.50	\$42.00
4	K	\$6.95	\$27.80
			\$66.32
			\$75.92
			\$42.00
			\$27.80
			\$212.04

Problem Solving

Strategies/Representation
 model (manipulatives - fraction bars / circles)
 circle (bar graph) area model
 table
 chart

Connections

- S and b are most popular balls - mode
- S and b end f and K had - minimum
- same number of votes
- f - most expensive ball
- K - least expensive ball
- find decimal and percent for each ball type
- Mr. Mitchell has \$12.96 remaining in his ball budget
- Relate to a similar problem and state math
- A new student would most likely select s or b
- $\frac{30}{90}$ select soccer balls - ratio

Mathematical Language

model
 circle graph
 bar graph
 table
 chart
 fractions $\frac{1}{3}$...
 decimals .333, .333
 percents 33.3%
 numerator
 denominator
 equivalent
 maximum
 Related Tasks
 ratio
 most
 least
 more/less than
 money notation
 \$.
 total
 cost
 key
 data
 most likely
 See Resource Binder

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Cherry Pies

Four classes of fifth graders are raising money for a field trip. The four classes bake cherry pies that are all the same size to sell at the George Washington birthday bake sale. Each cherry pie is cut in eight equal pieces. The fifth graders sell each piece of cherry pie for sixty-seven cents because that is how many years George Washington lived. At the end of the bake sale each class reports how much cherry pie was sold.

- The first class sold $4 \frac{2}{8}$ cherry pies.
- The second class sold $4 \frac{6}{8}$ cherry pies.
- The third class sold $1 \frac{9}{4}$ cherry pies.
- The fourth class sold $5 \frac{1}{2}$ cherry pies.

Which class sold the most pieces of cherry pie and what was the total amount of money the four fifth grade classes earned at the George Washington's birthday bake sale? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Cherry Pies Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link Everyday Mathematics, Units 5
 Common Core Standard(s) _____

Underlying Mathematical

Concepts
 fractional parts of a whole
 equivalent fractions
 addition multiplication
 money notation
 comparison
 number sense to \$108.54
 mixed numbers

Answer
 2nd class
 \$108.54

Possible Solution(s) Cherry Pies

class	amount sold	pieces sold
1st	$4\frac{2}{8}$	34
2nd	$\frac{46}{8}$	46
3rd	$\frac{19}{4} = 4\frac{3}{4} = 4\frac{6}{8}$	38
4th	$5\frac{1}{2} = 5\frac{4}{8}$	44

$34 + 46 + 38 + 44 = 162$ pieces

$$\begin{array}{r} 162 \\ \times 0.67 \\ \hline \$108.54 \end{array}$$

Problem Solving

Strategies/Representation
 Model (manipulatives - fraction circles)
 diagram/key - area models/
 circle graphs
 chart

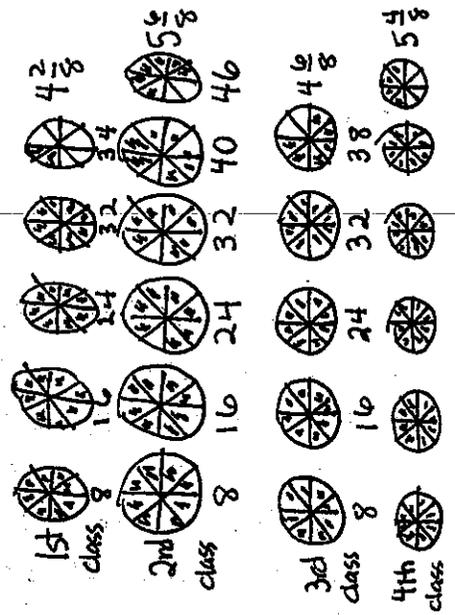
Mathematical Language

model total
 diagram cost
 key most/least
 area models common den.
 circle graphs numerator
 chart denominator
 fractions money notation
 mixed numbers \$.
 whole decimals.
 equivalent percents %
 $4\frac{2}{8}, 4\frac{4}{8}, \frac{19}{4}$ ordinal numbers
 1st, 2nd, 3rd, 4th

Related Tasks

See Resource Binder

- Connections
- $20\frac{3}{8} = 20\frac{1}{4}$ total pies
 - $\frac{46}{8} = 5\frac{7}{8}$ pie remaining
 - all pies have remaining pieces
 - 1st class sold least amount of pie
 - 2nd class sold most pie
 - 4th class sold 5 whole pies and 50% or .5 of 6th pie
 - Show decimals of pie remaining
 - $\frac{162}{8}$ pie is eaten
 - If all pie sold they would raise \$117.92



P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Lots of Cake

Mom bakes a chocolate flavored cake, a vanilla flavored cake, and a strawberry flavored cake. Each cake is the same size. Angela eats four-eighths of the chocolate cake, two-eighths of the vanilla cake and two-fourths of the strawberry cake. David eats one-fourth of the chocolate cake, three-eighths of the vanilla cake, and one-half of the strawberry cake. Who eats the most cake? How much of each cake is left? Show all of your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Lots of Cake Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link Everyday Mathematics, Unit 5
 Common Core Standard(s) _____

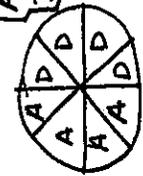
Underlying Mathematical Concepts

fractional parts of a whole
 improper / proper fractions
 comparing fractions with unlike denominators
 fraction notation
 addition

Answer 1
 Angela

Possible Solution(s)

chocolate vanilla strawberry cake cake cake



Key

A Angela
 D David

Angela eats $\frac{10}{8}$ or $1\frac{2}{8}$ or $1\frac{1}{4}$ cake
 David eats $\frac{9}{8}$ or $1\frac{1}{8}$ cake

CAKE	eaten	not eaten
chocolate	$\frac{9}{8}$	$\frac{2}{8}$
vanilla	$\frac{5}{8}$	$\frac{3}{8}$
strawberry	$\frac{4}{8} = 1$	0

Answer 2

Problem Solving Strategies/Representation

Model (manipulatives/fraction circles)
 diagram | key - area model/
 circle graph
 chart

Connections

- $\frac{19}{8} = 2\frac{3}{8}$ is total pieces of cake eaten
- Both kids ate 50% or .5 of the strawberry cake
- It appears that Angela likes vanilla cake the least and David chocolate cake
- It appears that David likes strawberry cake the most and Angela both chocolate and strawberry
- Angela ate $\frac{1}{4}$, 25%. 25% of vanilla cake
- $\frac{5}{8}$ slices remaining

Mathematical Language

Model whole
 diagram more/less than
 Key per
 chart decimal.
 circle graph percent %
 fractions $\frac{1}{2}, \frac{1}{4}, \frac{3}{8}, \dots$
 numerator total
 denominator
 improper fractions
 equivalent

Related Tasks

See Resource
 Binder

- Angela eats $\frac{1}{8}$ more cake than David
- Relate to a similar problem and state math
- David eats $1\frac{1}{8}$ cake but not a total cake of any flavor

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Coins in Grandpa's Pocket

Grandpa has twenty-four coins in his pocket. Grandpa gives Sam some clues about the total value of the coins in his pocket:

- One-sixth of the coins are quarters
- One-half of the coins are dimes
- Grandpa has an equal amount of nickels and pennies

Grandpa says Sam can have all the coins in his pocket if he discovers the correct total value of the coins. What is the total value of the coins in Grandpa's pocket that Sam should discover? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Coins in Grandpa's Pocket
 State Standard(s) Addressed
 Common Core Standard(s) 5.NF.1, 2
 Content Strand(s) Addressed Number Operations Fractions
 Program Link

Underlying Mathematical

- Concepts
- fractional part of a whole
- comparing fractions with unlike denominators
- value of coins
- money notation
- multiplication/addition

Answer
\$ 2.44

Possible Solution(s)

Coins in Pocket

Key
 D Dime
 Q Quarter
 P penny
 n nickel



$$\begin{array}{r} \frac{1}{2} \text{ of } 24 = 12 \\ \frac{1}{6} \text{ of } 24 = 4 \end{array}$$

coin	# of coins	value	total money
Q	4	\$1.00	\$1.00
D	12	\$1.20	\$2.20
N	4	\$.20	\$2.40
P	4	\$.04	\$2.44

Problem Solving

- Strategies/Representation
- area model - circle graph/bargraph
- table/chart
- model - fractions / circles

- Mathematical Language
- area model
- circle graph
- bargraph
- table/chart
- key
- data
- numerator
- denominator
- fractions $\frac{1}{2}$
- decimals .50
- percents 50%
- most/least/likely
- equivalent
- mode
- minimum
- maximum
- dime
- quarter
- nickel
- penny
- total
- ratio

Connections

- He had the most dimes in his pocket - maximum
- Had an equal amount of quarters, nickels, and pennies in his pocket but not the same total value
- find percent for each fractional part
- find decimal for each fractional part
- Solve with a different number of coins
- Show ratio $\frac{4}{24}, \frac{12}{24}, \frac{4}{24}$ are dimes
- relate to a similar problem and show math

Related Tasks

See Resource Binder

P/S	R/P	Com	Con	Rep	Ach/Level

Name: _____

Brownies

written by Bethanie, grade 5

Mrs. Wilson has 64 brownies on a plate. One half of her students eat one brownie. One fourth of the students keep one brownie for a snack. One eighth of the students keep one brownie to eat at lunch. Eight students keep one brownie to take home to eat. Are there any brownies left on Mrs. Wilson's plate? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Brownies Content Strand(s) Addressed Number Operations Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts
 fractional part of a whole
 comparing fractions with unlike denominators
 multiplication / addition
 number sense to 64

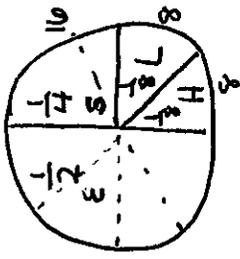
Problem Solving Strategies/Representation
 area model - circle graph / bar graph
 table / chart
 model - fraction bars / circles

Mathematical Language
 area model most-likely
 circle graph least-likely
 bar graph equivalent
 table mode
 chart maximum
 Key minimum
 fractions $\frac{1}{2}$... variables
 decimals .333 double
 percent 33.3% ratio
 denominator
 numerator
 data
 odd/even

Answer
 no brownies are left on the plate

Possible Solution(s) Students + Brownies

Key
 E eat 1 brownie
 S eat for snack
 L eat for lunch
 H take home
 T total students



$$E + S + H + L = T$$

$$32 + 16 + 8 + 8 = 64$$

$$\begin{array}{r} 8 \text{ eighths} \\ \times 8 \text{ } \times 8 \text{ students} \\ \hline 64 \text{ } 64 \end{array}$$

- Connections
- L and H are the minimum
 - E is the maximum
 - A new student would most likely eat a brownie
 - All even totals of brownies
 - Show decimal for each section
 - Show percent for each section
 - Relate to a similar problem and state math
 - twice as many students ate their brownie than took it home or ate at lunch
 - $\frac{32}{64}$ - ratio - eat a brownie

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

Favorite Sport

Sam asks the students in his school to select their favorite sport. One half of the students said their favorite sport was soccer. One-fourth of the students said their favorite sport was basketball. One-eighth of the students said their favorite sport was baseball. Twenty-seven students said their favorite sport was football. How many students does Sam ask to select their favorite sport? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task Favorite Sport Content Strand(s) Addressed Number Operations - Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts

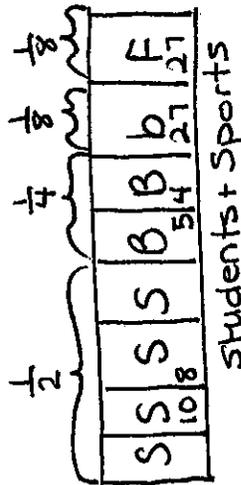
fractional part of a whole
 comparison of fractions with unlike denominators
 multiplication (addition)
 number sense to 216

Problem Solving Strategies/Representation

area model/circle graph/bar graph
 table/chart
 model - fraction bars/circles

Mathematical Language
 area model total
 circle graph variables
 bar graph mode
 table data
 chart minimum
 key maximum
 fractions $\frac{1}{4}$... equivalent
 decimals .25
 percents 25%
 numerator
 denominator
 most/least likely

Possible Solution(s)



$$S + B + F = T$$

$$108 + 54 + 27 + 27 = 216$$

$$\begin{array}{r} 27 \\ \times 8 \\ \hline 216 \end{array}$$

Key
 S Soccer
 B basketball
 b baseball
 f football
 T Total students

Connections

- Soccer is the mode
- baseball is equivalent to football
- show percent of each ball
- show decimal of each ball
- Football is least popular
- Relate to a similar problem and state math
- A new student would most likely pick soccer
- $\frac{9}{8} = \frac{1}{4}$ $\frac{1}{4} < \frac{1}{2}$
- show a ratio $\frac{108}{216}$ select Soccer

Related Tasks

See Resource Binder

P/S	R/P	Com	Con	Rep	Ach/Level

New Books for the Library

The Washington School librarian wants to order some new books for the library. The librarian asks the Washington School students what types of books she should order. One-fourth of the students want mystery books. One-fourth of the students want adventure books. One-fourth of the students want non fiction books about animals. One-eighth of the students want sports books. Fifty-seven students want non fiction books about countries. How many students does the Washington School librarian ask about ordering new books for the library? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task New Books for the Library Content Strand(s) Addressed Number + Operations + Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts
 fractional part of a whole
 multiplication / addition
 number sense to 456
 comparing fractions with unlike denominators

Problem Solving Strategies/Representation
 model (manipulatives - fraction bars / circles)
 circle graph / bar graph (area model table chart)

Mathematical Language
 model equivalent fractions
 area model data more/less than
 circle graph total
 bar graph minimum
 table maximum
 chart fractions
 fractions numerator
 denominator
 decimals
 percents

Possible Solution(s)
 types of books

114	114	114	57	57
1/4	1/4	1/4	1/8	1/8
M	A	A	S	C

$$\begin{array}{r} 57 \\ \times 8 \\ \hline 456 \end{array}$$

$$M + A + A + S + C = T$$

$$114 + 114 + 114 + 57 + 57 = 456$$

Answer
 456 students

- Connections
- There was a "tie" between M, A, and a - maximum
 - There was a "tie" between S and C - minimum
 - Show percent for each book type
 - Show decimal for each book type
 - $M > S < A$ $S + C = M$
 - Relate to a similar problem and state math
 - $\frac{114}{457}$ select mystery ... (ratio)

Key

M mystery
 A adventure
 a animals
 S sports
 C countries
 T total
 students

Related Tasks
 See Resource Binder

P/S	R/P	Com	Con	Rep	Ach/Level

Name _____

A Field Trip

Written by Kaylee, grade 5

The fifth graders vote to determine where they want to go on a field trip. One-third of the fifth graders want to go bowling. Two-sixths of the fifth graders want to go to the zoo. One-sixth of the fifth graders want to go to a museum. Sixteen students want to do activities at the town park. How many fifth graders vote to determine where they want to go on the field trip? Show all your mathematical thinking.

Preliminary Planning Sheet for a Mathematics Portfolio Piece/Task

5

Title of Task A Field Trip Content Strand(s) Addressed Number+Operations Fractions
 State Standard(s) Addressed _____ Program Link _____
 Common Core Standard(s) 5.NF.1, 2

Underlying Mathematical Concepts

fractional part of a whole
 comparisons of fractions with unlike denominators
 multiplication / addition
 number sense to 96

Problem Solving Strategies/Representation

area model / circle graph / bar graph
 table / chart
 model - fraction bars / circles

Answer
 96 students

Possible Solution(s)

Field trip

B	Z	M	P
32	32	16	16

$$\frac{1}{3} = \frac{1}{3} + \frac{1}{6} + \frac{1}{6}$$

$$B + Z + M + P = T$$

$$32 + 32 + 16 + 16 = 96$$

$$\begin{array}{r} 16 \\ \times 6 \\ \hline 96 \end{array}$$

Connections

- equivalent number of students picked bowling and zoo
- equivalent number of students picked museum and park
- even number of students in each category
- show decimal and percent for each category

Relate to a similar problem and state math

Relate to a similar problem and state math

$$\frac{1}{3} = \frac{2}{6}$$

ratio $\frac{32}{96}$ select bowling...

Key

B	bowling
Z	zoo
M	museum
T	total students
P	park

place Fractional part students total fractional part

bowling	$\frac{1}{3} = \frac{2}{6}$	$\frac{2}{6}$
zoo	$\frac{1}{3} = \frac{2}{6}$	$\frac{2}{6}$
museum	$\frac{1}{6}$	$\frac{1}{6}$
park	$\frac{1}{6}$	$\frac{1}{6} = 1$

Mathematical Language
 area model
 circle graph
 bar graph
 table
 chart
 key
 fractions $\frac{1}{3}$... ratio
 decimals .333
 percent 33 1/3 %...
 denominator
 numerator
 data

Related Tasks

See Resource Binder



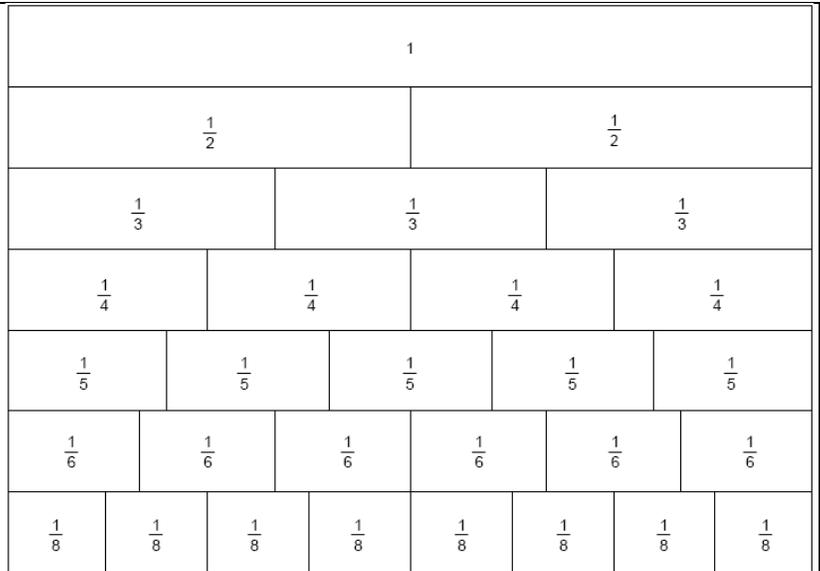
GRADE 5 MATH: STUFFED WITH PIZZA
SUPPORTS FOR ENGLISH
LANGUAGE LEARNERS

GRADE 5 MATH: STUFFED WITH PIZZA

Supports for ELLs

Title: Stuffed With Pizza	Grade: 5
Linguistic Access: <p>In these supportive materials, a distinction between the vocabulary and the language functions is needed to provide entry points to the math content. Both need to be clarified to ensure comprehension and to avoid misunderstanding. This can be done by introducing and/or reviewing the most essential vocabulary and language functions in context and with concrete models, when applicable, in order for English Language Learners (ELLs) to better understand the meaning of the terms. The following vocabulary/language functions are suggested:</p> <p><u>Vocabulary Words/Phrases:</u></p> <p>Tier I (non-academic language): stuffed, ate, autumn, hiking, rock climbing, left over, earn, bowling</p> <p>Tier II (general academic language): show your mathematical thinking, greater than/less than, more than/less than, the most, per, budget, to determine</p> <p>Tier III (math technical language and concepts that must be carefully developed): one-fourth, one-half, three-eighths, five-eighths (etc.), percent, total cost, total value</p> <p><u>Language Functions:</u> show, explain, determine</p>	
Content Access: <p>To provide content access to ELLs, it is important that they are familiar with the concepts of fractional parts of a whole, equivalent fractions, fraction addition with unlike denominators, and understand the relationship among fractions, decimals, and percents. ELLs should already be familiar with the concept and interpretation of fractions and their symbolic representation.</p> <p><u>Fraction Strips</u></p> <p>Provide ELLs with a set of fractions strips and allow them to make sense of them.</p> <p>Fraction strips usually look like the one that is shown below (to the right of the bulleted list of possible prompts).</p> <p>Write a set of guiding prompts that may help ELLs make sense of the fraction strips. The following is an example of a possible set of prompts:</p>	

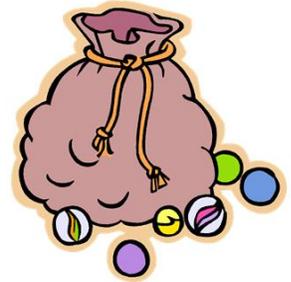
- How many halves does it take to make a whole?
- How many fourths does it take to make a half?
- How many fourths does it take to make a whole?
- Extend your thinking to the $\frac{1}{8}$ strip and explore its relationships with the other fractions.
- What are all of the equivalent fractions that you can find in the diagram?



Fractions have a reputation for being confusing, and there are many reasons why that is the case. However, one very important area about fractions that requires explicit teaching for ELLs is that a fraction can be used to describe both the parts of the whole and/or the parts of a set of discrete objects. This phenomenon makes it necessary to provide examples and manipulatives that allow ELLs to form this concept.

Example:

Maria has a bag with 12 marbles. She gave half of the marbles to Pedro, and then, from what was left, she gave one-third to Jose. How many marbles does she have left?



Ask ELLs to identify the “whole” in this situation. If necessary, point out that in this case the whole refers to the set of 12 marbles. This is a different context for the use of fractions and it should not be assumed that ELLs have made this connection on their own.

Problems involving a pizza (or any type of pie) are usually common in the context of fractions. In most situations, a pizza represents a whole. However, there might be situations where the “whole” is defined as a set of two or more pizzas. This can be particularly confusing. Perhaps providing additional situations might remind ELLs to always identify what is being defined as a whole by the situation in the problem.

Example:

Josefina ordered three pizzas. Then, she invited Jose to eat with her. Together they ate two-thirds of the order. How many pizzas are left?

In this situation, the original order (a set of three pizzas) is defined as the whole.



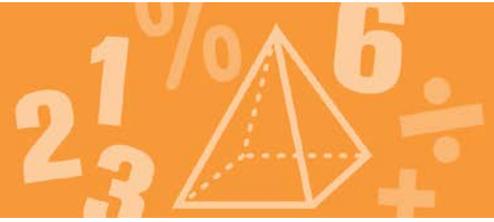
Scaffolds and Resources:

- Activate prior knowledge of the concepts of fraction and equivalent fractions by relating these mathematical ideas to students' everyday experiences. For example:
 - Teacher uses symbolic representations—such as pictures, pictographs, and diagrams—to assist ELLs in making connections between what they already know and the new ideas presented.
- Include, as much as possible, nonlinguistic representations (e.g., illustrations, photos, models, real objects, pictures) to assist ELLs in understanding the context of the problems selected.
- Review explicitly the academic language necessary in this unit and model the meanings of the words/phrases, if needed (e.g., numerator, denominator, equivalent, whole). It is recommended that:
 - Teacher models desired academic language in context to develop second language in addition to math content.
 - Teacher uses physical objects to facilitate students' understanding of the required mathematical concepts (e.g. manipulatives such as fraction wheels, whole-fraction bars).
 - Teacher uses concept organizers (e.g., the Frayer Model, which describes characteristics, examples, and non-examples of key concepts/phases) to front-load key content and functional academic language in context.
- Organize the tasks to maximize opportunities for ELLs to engage in math discourse. Therefore, it is recommended that:
 - Teacher allows students to work collaboratively in pairs or triads and to justify their decisions to peers.
 - Teacher allows ELLs to use their language resources, including their native language, gestures, drawings, etc. to convey their understandings.
 - Teacher comes prepared with pivotal questions that would move the mathematical discourse into the understanding of the essential or important mathematics embedded in the task.
 - Teacher gives appropriate wait time for ELLs to respond.
 - Teacher gives the opportunity for ELLs to clarify their statements using different expressions.
 - Teacher uses paraphrases and “re-voicing” (reformulation of students' statements using appropriate math terminology or syntax).
- Facilitate a metacognitive approach to reading a math problem by encouraging students to monitor their understanding of the situation presented. This might include asking ELLs to:
 - Listen to the problem being read.
 - Read the problem by themselves or in small groups.
 - Discuss it with a partner.

- Underline relevant information.
- Identify what the problem is asking and what students need to do to solve it.

One good strategy is Reciprocal Teaching, where students follow a structured dialogue that involves questioning, summarizing, clarifying, and predicting.

- On page 33, there are some instructional implications that teachers may use to scaffold ELLs knowledge while introducing the concepts in the tasks. Although several are listed, we recommend the ones below, which have been slightly modified:
 - Use games requiring the use of fractions with and without like denominators (e.g., “I have ... Who has ...?”).
 - Have centers or “clinics” available for investigation and practice of those fractional concepts needed.



GRADE 5 MATH: STUFFED WITH PIZZA
SUPPORTS FOR STUDENTS
WITH DISABILITIES

GRADE 5 MATH: Stuffed with Pizza

Instructional Supports for Students with Disabilities using UDL Guidelines

Guideline 1: Provide options for perception

Learning is impossible if information is imperceptible to the learner, and difficult when information is presented in formats that require extraordinary effort and assistance. To reduce barriers to learning, it is important to ensure that key information is equally perceptible to learning by:

1. **Providing the same information through different modalities** (e.g., through vision, hearing, or touch)
2. **Providing information in a format that will allow for adjustability by the user** (e.g., text that can be enlarged, sounds that can be amplified)

(CAST (2011). *Universal Design for Learning Guidelines version 2.0*. Wakefield, MA: Author.)

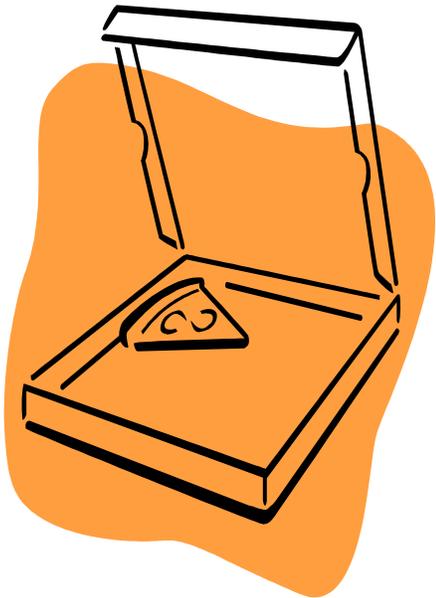
❖ **Offer ways of customizing the display of information.**

- Provide an image of pizza.
- Distinguish names by applying bold type-face
- Provide larger font size and additional spacing between lines

- ### ❖ **Offer alternatives for visual information.** Read aloud and/or record text, Stuffed with Pizza, to allow students multiple opportunities to hear text read aloud.

Name _____

Stuffed with Pizza



Tito and **Luis** are stuffed with pizza! **Tito** ate one-fourth of a cheese pizza. **Tito** ate three-eighths of a pepperoni pizza. **Tito** ate one-half of a mushroom pizza. **Luis** ate five-eighths of a cheese pizza. **Luis** ate the other half of the mushroom pizza. All the pizzas were the same size. **Tito** says he ate more pizza than **Luis** because **Luis** did not eat any pepperoni pizza. **Luis** says they each ate the same amount of pizza. Who is correct? Show all your mathematical thinking.

Guideline 2: Provide options for language, mathematical expressions, and symbols

The semantic elements through which information is presented – the words, symbols, numbers, and icons— are differently accessible to learners with varying backgrounds, languages, and lexical knowledge. To ensure accessibility for all, key vocabulary, labels, icons, and symbols should be linked to, or associated with, alternated representations of their meaning (e.g., an embedded glossary or definition, a graphic equivalent, a chart or map). Idioms, archaic expressions, culturally exclusive phrases, and slang, should be translated.

(CAST (2011). *Universal Design for Learning Guidelines version 2.0*. Wakefield, MA: Author.)

- ***Pre-teach vocabulary and symbols, especially in ways that promote connection to learners experiences and prior knowledge:***
- ***Promote understanding across languages by making all key information in the dominant language (e.g., English) also available in native languages for English Language Learners and in American Sign Language for learners who are deaf***
 - Stuffed with Pizza: stuffed, pizza, pepperoni, cheese, mushroom, cheese pizza, pepperoni pizza, and mushroom pizza
 - Autumn Fun: outdoors, hiking, bike riding, and rock climbing
 - Eating Pie: pie, cherry, apple, blueberry, cherry pie, apple pie, blueberry pie, and left-overs
 - Pizza for a Party: planning a party, topping, favorite topping, pepperoni, cheese, sausage, mushroom, mushrooms
 - New Balls for Recess: recess, soccer, soccer balls, basketballs, footballs, kickballs, order, and budget
 - Cherry Pies: raising money, field trip, cherry pies, George Washington, equal, and bake sale
 - Lots of Cake: vanilla, chocolate, strawberry, chocolate flavored cake, vanilla flavored cake, and strawberry flavored cake
 - Coins in Grandpa's Pocket: coin, quarter, dime, nickel, pennies, Grandpa, clue, and pocket
 - Brownies: brownie, and snack
 - Favorite Sport: sport, favorite, soccer, basketball, and baseball
 - New Books for the Library: librarian, mystery, adventure, nonfiction, mystery books, adventure books, and nonfiction books
 - A Field Trip: field trip, bowling, zoo, museum, and town park
- ***Illustrate through multiple media.*** Present key concepts in one form of symbolic representation (e.g., math equation or expository text) with an alternative form (e.g., an illustration, dance/movement, diagram, table, model, video, comic strip, storyboard, photograph, animation, physical or virtual manipulatives):
 - Fraction Tiles/ Virtual Manipulatives
Fraction tiles are a good way for students to explore and understand how parts make up a whole. Teachers and parents can utilize this resource when giving a lesson on fractions with students.
http://www.abcya.com/fraction_tiles.htm

- Review of equivalent fractions
A computer game that involves the use of pizza slices and their equivalent fraction numerical representations. Students use the computer mouse to move pizza slices to their correct equivalent places on a pizza.
<http://www.softschools.com/math/fractions/games/>

- Fraction Pizza using Real Pizza Boxes
This activity involves stopping by local pizza shops and asking for them to donate pizza boxes for this activity.
<http://teachers.net/lessons/posts/1640.html>



MrNussbaum.com offers dozens of interactive games specifically designed to pinpoint one or several essential concepts taught in elementary years, and dozens of research themes designed to make the process of gathering information interactive and enjoyable.
<http://www.mrnussbaum.com/tonyfraction.htm>

- Learn how to recognize fractions by breaking shapes into equal parts
This domain of the Common Core focuses on understanding fractions as numbers. Students learn how to build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. They also learn how to compare fractions and to use decimal notation.
<http://www.learnzillion.com/lessons/75>

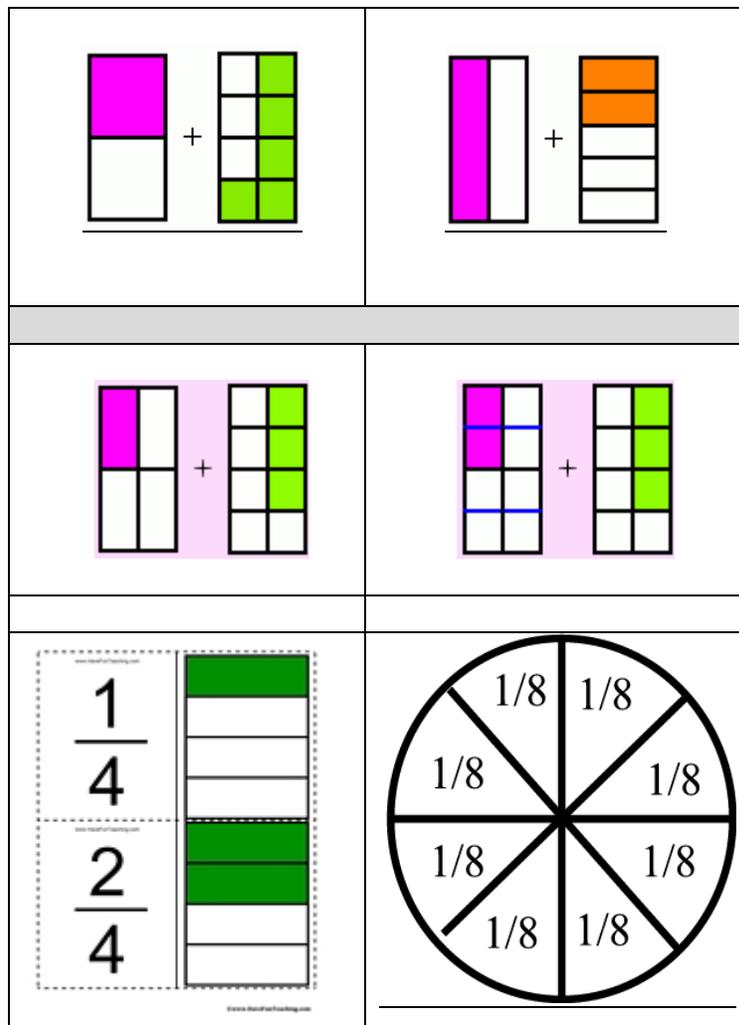
- Review and Practice: Write fractions w numerator of 1 using shapes
In this lesson, with audio component, students will learn to write fractions with a numerator of one by looking at shapes that are broken into equal shapes
<http://www.learnzillion.com/lessons/76>

- Review and Practice: Fractions
Students work individually. IXL tracks student's score, and the questions increase in difficulty as the student improves!
<http://www.ixl.com/math/grade-5/fractions-review>

- Review and Practice: Equivalent Fractions
Students will identify the missing number to make the fraction equal.
<http://www.ixl.com/math/grade-5/equivalent-fractions>

- Review and Practice: Fractions and Mixed Numbers
Convert between improper fractions and mixed numbers
<http://www.ixl.com/math/grade-5/convert-between-improper-fractions-and-mixed-numbers>

- Review and Practice: Fractions and mixed numbers
Compare fractions and mixed numbers
<http://www.ixl.com/math/grade-5/compare-fractions-and-mixed-numbers>
- Review and Practice: Comparing Unlike Fractions
Compare fractions to identify if they are *greater than*, *less than* or *the same*
<http://www.aaamath.com/fra43cx2.htm>
- Adding Unlike Fractions using Number Lines
An on-line tutorial that offers instruction and practice in adding unlike fractions. All examples are pictured with number lines
<http://www.visualfractions.com/AddUnlike/addunlike.html>
- Provide samples of Fraction Cards



- Printable Fraction Flash Cards
<http://www.havefunteaching.com/flash-cards/math/fraction-flash-cards.pdf>

- Adding Fractions with Different Denominators
An interactive math lesson about adding fractions with different denominators
<http://www.aaamath.com/fra66kx2.htm>

Guideline 3: Provide options for comprehension

The purpose of education is not to make information accessible, but rather to teach learners how to transform accessible information into useable knowledge. Individuals differ greatly in their skills in information processing and in their access to prior knowledge through which they can assimilate new information. (CAST (2011). *Universal Design for Learning Guidelines version 2.0*. Wakefield, MA: Author.)

- ❖ **Activate or supply background information by anchoring instruction and pre-teaching critical prerequisite concepts:** *decimals, denominators, equivalent fractions, fractions, numerators, percents, mixed numbers, base ten number system, patterns, sums, products, differences, estimation, factors, more than, less than, quotient, and total.*

Guideline 6: Provide options for executive functions

One of the limits of executive functions is that it is imposed by so-called working memory. This ‘scratch pad’ for maintaining chunks of information where they can be accessed as part of comprehension and problem-solving is very limited for any learner and even more severely limited for many learners with learning and cognitive disabilities. As a result, many such learners seem disorganized, forgetful, and unprepared. (CAST (2011). *Universal Design for Learning Guidelines version 2.0*. Wakefield, MA: Author.)

- ❖ **Facilitate managing information and resources.** Provide graphic organizers and templates for data collection and organizing information:

Stuffed with Pizza		
What questions am I being asked to answer? Write them down.	What <u>strategy or strategies</u> can I use to solve the math word problem and answer <u>all</u> the questions? Write them down.	How can I show <u>all</u> my mathematical thinking and proof? Write it down.

- ❖ **Guide appropriate goal –setting.** Review rubric and provide a checklist to support students mathematical thinking and problem-solving strategies:

Checklist		
After reading each question, check <u>Yes</u> or <u>No</u>	Yes	No
Did I read the math problem several times?		
Did I know what the problem was asking me to solve?		
Did I <u>label</u> my work correctly?		
Did I check <u>all</u> my computations?		
Did I show <u>how</u> I solved the problem?		
Did I show <u>all</u> my mathematical thinking?		
Did I justify and defend my answers?		
Did I answer <u>all</u> the questions?		

Guideline 9: Provide options for self-regulation

While it is important to design the extrinsic environment so that it can support motivation and engagement, it is also important to develop learners' intrinsic abilities to regulate their own emotions and motivations. While many individuals develop self-regulatory skills on their own, either by trial or error or by observing successful adults, many others have significant difficulties in developing these skills. A successful approach requires providing sufficient alternatives to support learners with very different aptitudes and prior experience to effectively manage their own engagement and affect. (CAST (2011). *Universal Design for Learning Guidelines version 2.0*. Wakefield, MA: Author.)

- ❖ **Promote expectations and beliefs that optimize motivations** by increasing the length of on-task orientation in the face of distractions. Teachers should be aware of the personal needs and styles of their students. Time management needs must be addressed. A time-on task chart can be created:

My Time-on Task Chart	
Task	Time
Read problem	<u>4</u> minutes
Highlight important information	<u>3</u> minutes
Create a chart	<u>4</u> minutes
Set up problem	<u>2</u> minutes
Solve problem	<u>2</u> minutes