GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS

BENCHMARK PAPERS WITH RUBRICS

This section contains benchmark papers that include student work samples for each of the tasks in the Ratios and Proportional Relationships assessment. Each paper has descriptions of the traits and reasoning for the given score point, including references to the Mathematical Practices.

NYC Grade 6 Assessment 1
Elevator Task
Benchmark Papers

1. Giovanni is visiting his grandmother who lives in an apartment building on the 25th floor. Giovanni enters the elevator in the lobby, which is the first floor in the building. The elevator stops on the 16th floor. What percentage of 25 floors does Giovanni have left to reach his grandmother's floor? Use pictures, tables or number sentences to solve the task. Explain your reasoning in words.

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3 Points

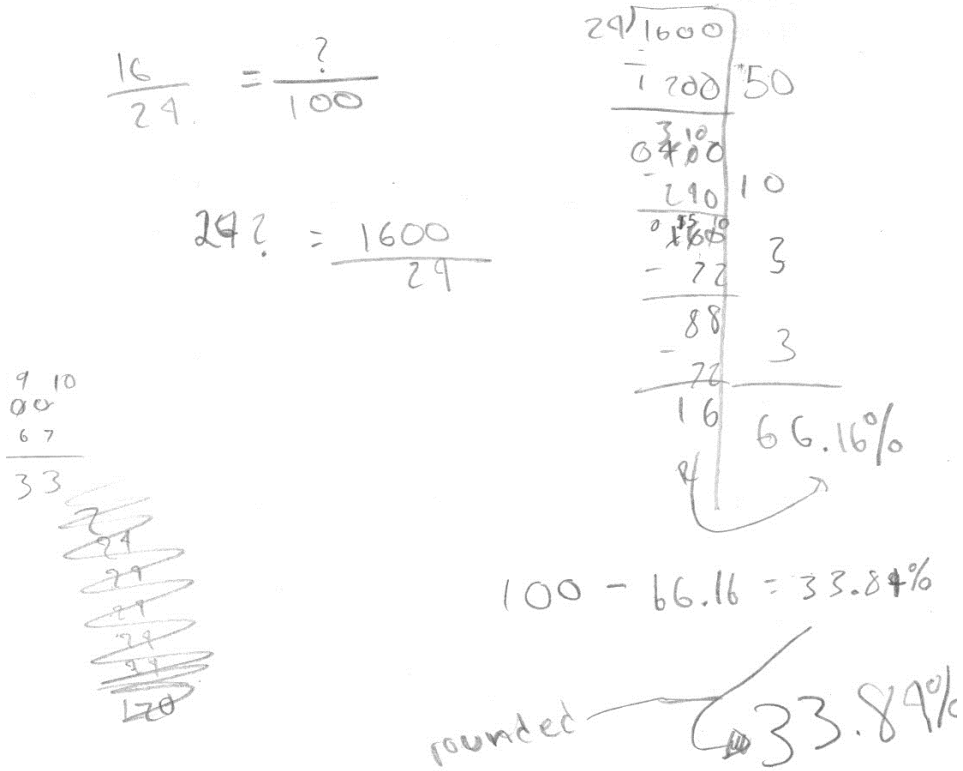
The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratio and percent. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with part-whole, ratio and percent). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a ratio and/or decimal and percent. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

a. Understanding that the whole is 25 and either 16 or (25 – 16) floors is a part of the whole involved. Some students may consider a "missing 13th floor". In that case, the whole is 24 and either 15 or (24 – 15) is a part of the whole involved.
b. Recognition of the need to determine how many "out of 100".
    i. Using a fraction and its conversion to an equivalent fraction, possibly by simplifying first.
    ii. Converting to decimal then to percent.
    iii. Creating and reasoning from a grid representation of the context
c. Recognition of the need either to work with 9/25 (or 9/24) or to subtract from 100% after changing 16/25 (or 15/24) to percent.



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2 Points

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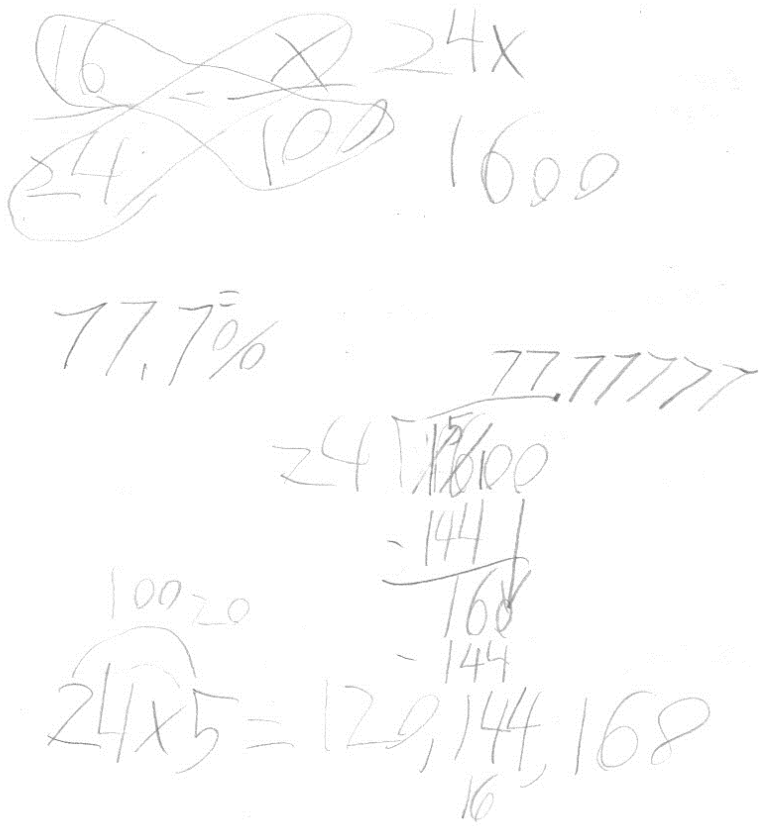
The reasoning used to solve the parts of the problem may include:

a. Understanding that the whole is 25 (or 24) and either 16 (or 15) floors is a part of the whole but failing to recognize the need to use 9/25 (or 9/24) or to subtract from 100% after changing 16/25 (or 15/24) to percent.
b. Recognition of the need to work with 9/25 (or 9/24), but failing to change the ratio into a percent.



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1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratio and percent or partial answers are evident. Explanations are incorrect, incomplete or not based on work shown. Accurate reasoning processes demonstrate the Mathematical Practices, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with part-whole, ratio and percent). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem as a ratio and/or decimal and percent. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio notation and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

a. Forming the ratio 16/25 (or 15/24), and trying but failing to change the ratio into a percent.
b. Forming the ratio 9/16 (or 9/24) and changing that ratio to a percent.

Answer: 15%

$\frac{16}{24}$

$$\begin{array}{r} 1.5 \\ 16\overline{)24.0} \\ -16 \\ \hline 80 \\ -80 \\ \hline 0 \end{array}$$

$$\begin{array}{r} {}^{1}16 \\ *\ 2 \\ \hline 32 \end{array}$$

$$\begin{array}{r} 416 \\ *\ 7 \\ \hline 112 \end{array}$$

$$\begin{array}{r} {}^{3}16 \\ *\ 5 \\ \hline 80 \end{array}$$

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2. Pianos and pipe organs contain keyboards, a portion of which is shown below.

a) What is the ratio of black keys to white keys in the picture above?

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

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3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve each part of the problem is stated, as is the work used to find ratios, proportions, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear.

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The reasoning used to solve the parts of the problem may include:

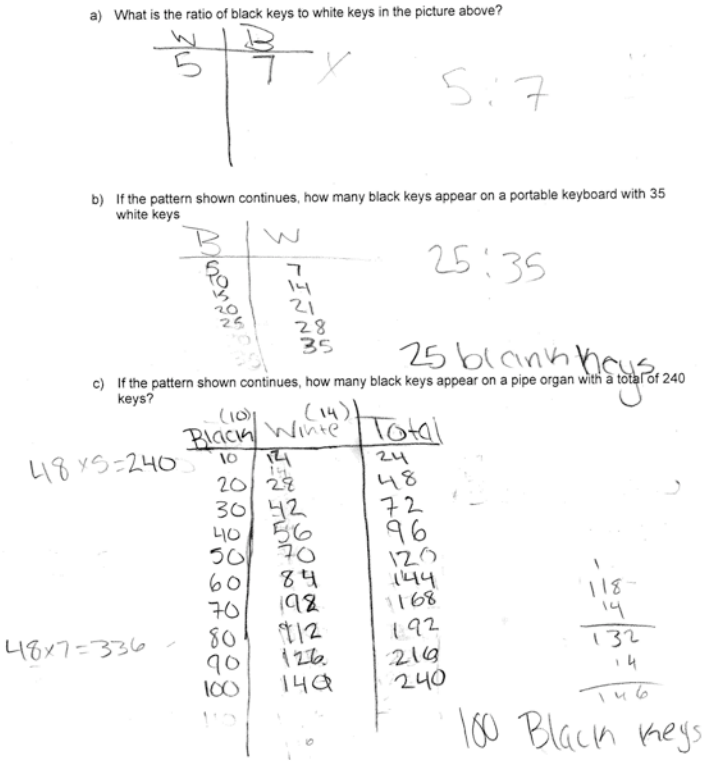
a. Scaling up the 5:7 ratio in fraction form to a denominator of 35, signifying 35 white keys.

5	10	15	20	25
7	14	21	28	35

b. Scaling up the 5:7 ratio in tabular form.

c. Using a proportion or proportional reasoning (e.g., 35 white keys is 5 times 7 white keys, so I can multiply 5 by 5 black keys to find the number of black keys).

d. Recognizing the part-to-whole nature of part c as 5:12 and employing any of the above techniques.



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2 Points

The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

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The reasoning used to solve the parts of the problem may include:

5	10	15	20	25
7	14	21	28	35

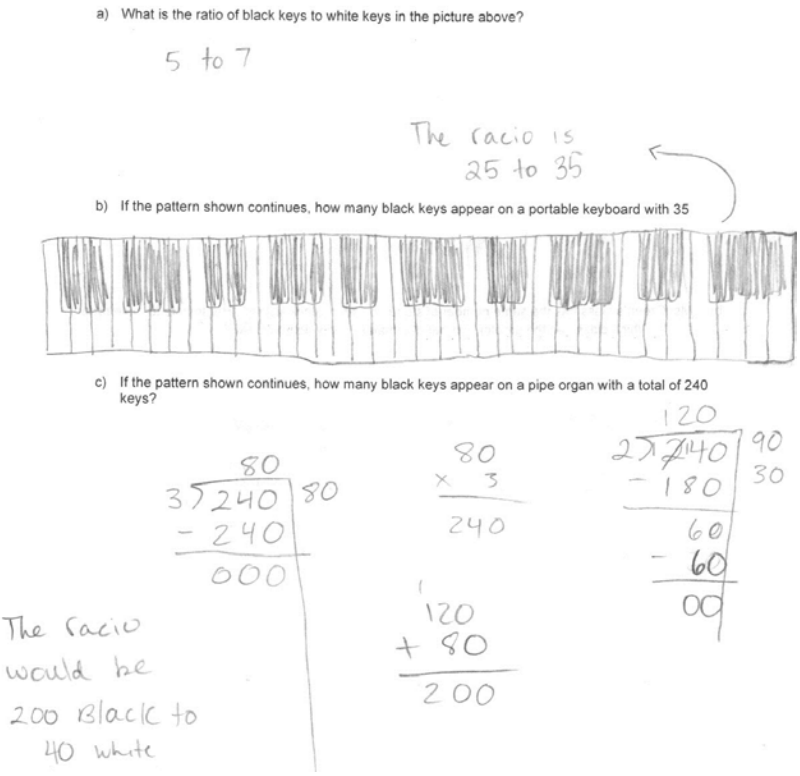
a. Scaling up the 5:7 ratio in fraction form to some denominator, but failing to stop at 35, signifying 35 white keys.

b. Scaling up the 5:7 ratio in tabular form, but failing to stop at 35.

c. Reversing white and black keys, but maintaining the scaling to 35 white keys as indicated by labels.

d. Using a proportion or proportional reasoning (e.g., 35 white keys is 5 times 7 white keys), but failing to then multiply 5 by 5 black keys to find the total number of black keys.

e. Recognizing the part-to-whole nature of part c as 5:12 and employing any of the above techniques or errors.



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The reasoning used to solve the parts of the problem may include:

a. Identification of the 5:7 ratio in some form (including tabular).

b. Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.

c. Failure to attempt at least two parts of the problem.

a) What is the ratio of black keys to white keys in the picture above?

w	b
7	5

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys

w	b
7	5
14	10
21	15
~~21~~ 28	
35	20
90 ~~55~~	35
55	49
20	
75	77

+14

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

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3.

a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.

b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

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The reasoning used to solve the parts of the problem may include:

a. Scaling up the 3:2 ratio in fraction form to a numerator of 18, signifying 18 girls. Recognizing 18 + 12 as the whole, and 2:1 as the ratio of girls-to-whole in the other class, and following another scaling-up process.

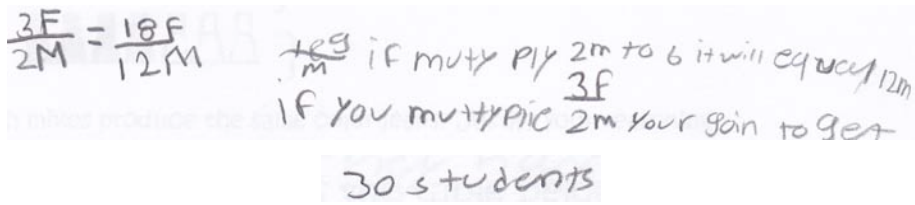
3	6	9	12	15	18
2	4	6	8	10	12

b. Scaling up the 3:2 ratio in tabular form; scaling up the 2:1 ratio in a similar fashion, to a total of 30 students.

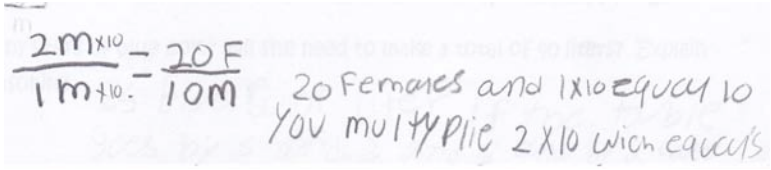
c. Using a proportion or proportional reasoning (e.g., 18 girls is 6 times 3 girls, so I can multiply 6 by 2 boys to find the number of boys). Using a part-whole fraction to form a proportion for the second class.

d. Recognizing that a 3:2 ratio is being compared with a 2:1 ratio for the same number of students. As a result, the ratio that represents a larger fraction also represents the class with the larger number of girls, since the ratio is female-to-male in both instances.

a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.



b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?



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The reasoning used to solve the parts of the problem may include:

a. Scaling up the 3:2 ratio in fraction or tabular form, but failing to stop at a numerator of 18, signifying 18 girls. Recognizing 2:1 as the ratio of girls to whole in the other class, and following another scaling up process (possibly incorrectly).

b. Recognizing the need for a whole, but determining it with faulty reasoning or scaling part b to 18 girls rather than 30 students.

c. Using a proportion or proportional reasoning (e.g., 18 girls is 6 times 3 girls, so I can multiply 6 by 2 boys to find the number of boys). Incorrectly using a part-part fraction to form a proportion for the second class.

d. Recognizing that a 3:2 ratio is being compared with a 2:1 ratio for the same number of students; failing to determine the ratio that represents the larger fraction.

a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show how you determined your answer. Explain your reasoning in words.

	F	M	T
	3	2	5
x6	18	12	30

12 boys

b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

	F	M	T
	2	1	3
x9	18	9	27

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1 Point

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Some evidence of reasoning is demonstrated either verbally or symbolically, but may be based on misleading assumptions, and/or contain errors in execution. Some work is used to find ratio and scaling or partial answers are evident. Explanations are incorrect, incomplete or not based on work shown. The student may make some attempt at using some mathematical practices.

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The reasoning used to solve the parts of the problem may include:

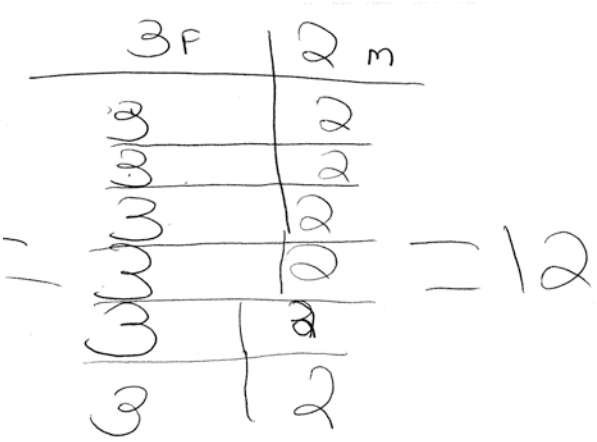
a. Identification of the 3:2 ratio in some form (including tabular).

b. Some attempt to scale, but failure to maintain the ratio, typically by reverting to addition.

c. Failure to recognize that ratios are being compared in part b; attempting instead to compare by using the 3 from Mr. Copper's class with the 2 from Ms. Green's class (or 2 vs. 1).

d. Failure to attempt at least one part of the problem

a) Mr. Copper's class has a female student-to-male student ratio of 3:2. If Mr. Copper's class has 18 girls, how many boys does he have? Show how you determined your answer. Explain your reasoning in words.



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b) Ms. Green's class has the same number of students as Mr. Copper's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

mr. Copper has more female
because he has a bigger nemb
2 multiply

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4. Use the recipe shown in the table to answer the questions below. Use pictures, tables or number sentences to solve the task.

Grandma's Recipe for Sugar Cookies
1 ½ cups butter
2 cups sugar
4 eggs
¾ teaspoon baking powder
1 ¼ cups flour
¼ teaspoon salt

a) How many cups of sugar are needed for each egg? How do you know?

b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

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The reasoning used to solve the parts of the problem may include:

a. A ratio of cups of sugar to each egg is formed and simplified or scaled down to a denominator of 1.
b. Scaling down the 2:4 ratio in tabular form.

2	1	½
4	2	1

c. Recognizing the phrase "three times as much baking powder as salt" as a 3:1 ratio.
d. Forming a ¾ : ¼ ratio; possibly scaling that ratio up to 2 ¼ : ¾ on the basis of the "three times as much" language.
e. Drawing a picture and reasoning from the picture for either or both parts.

¼	¼	¼
¼	¼	¼
¼	¼	¼
¼	¼	¼

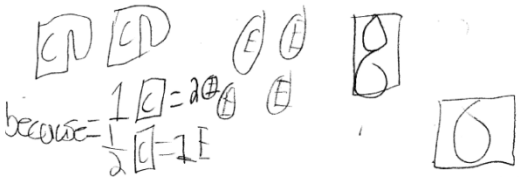
Baking Powder

¼
¼
¼
¼

Salt

3 teaspoons baking powder : 1 teaspoon salt

a. How many cups of sugar are needed for each egg? How do you know?



b. Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

3/4 : 1/4

I know because baking powder comes first then salt then you just put the recipe volume with baking powder first then salt.

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The reasoning used to solve the parts of the problem may include:

a. A ratio of cups of sugar to each egg is formed but not scaled down to a denominator of 1.
b. A ratio of eggs to cups of sugar is formed and scaled down to a denominator of 1.
c. Forming a ¾ : ¼ ratio; but incorrectly attempting to multiply by 3 on the basis of the “three times as much” language.
d. Drawing a picture for both parts, but reasoning incorrectly from the picture for one part.

a. How many cups of sugar are needed for each egg? How do you know?

2 cups because if you divide 4 eggs into 2 cups of sugar it will be 2 eggs per cup.

b. Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

3/4 teaspoon : 1/4 teaspoon.

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The reasoning used to solve the parts of the problem may include:

a. Expressing a correct ratio, with no explanation.
b. Forming the ratio of eggs to cups of sugar, but not scaling down to a denominator of 1.
c. Forming a ratio from two ingredients, only one of which is baking powder or salt; however, correctly placing the baking powder or salt in the ratio.
d. Drawing a picture, but reasoning incorrectly from the picture for both parts.

a) How many cups of sugar are needed for each egg? How do you know?

1/2 a cup of Surgar

b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain your reasoning in words.

~~1/12~~ 1 1/2 : 1 1/4

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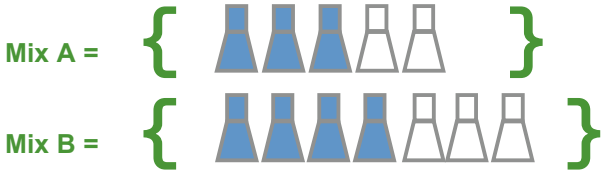
5. Fashion designers are trying to decide on just the right shade of blue for a new line of jeans. They have several bottles of fabric color, some with blue color and some with white color. They plan to mix these together to get the desired color.



a) Will both mixes produce the same color jeans? Justify your reasoning.

b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16

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3 Points

The response accomplishes the prompted purpose and effectively communicates the student's mathematical understanding. The student's strategy and execution meet the content (including concepts, technique, representations, and connections), thinking processes and qualitative demands of the task. Minor omissions may exist, but do not detract from the correctness of the response.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, scaling, and partial answers to problems. Minor arithmetic errors may be present, but no errors of reasoning appear. Complete explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

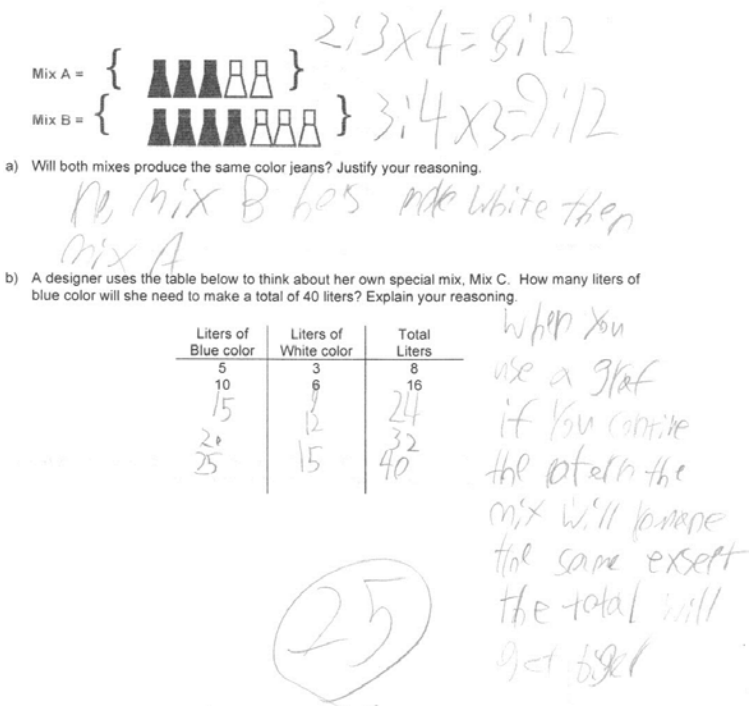
Justification may include reasoning as follows:

a. Forming unit rates and indicating that the unit rates differ or are not equivalent.
b. Forming ratios and indicating that the ratios differ or are not equivalent.
c. Building a table of values to a common number and noting, e.g., that one mix uses 9 blue and 6 white while the other differs, using 8 blue and 6 white, so the first is “bluer”.

3	6	9
2	4	6

4	8
3	6

d. Extending the table to a total of 40 and correctly identifying the number of blue liters.
e. Using a proportion or proportional reasoning (e.g., 40 total liters is 5 times 8, so I can multiply 8 by 5 to find the number of blue liters).



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2 Points

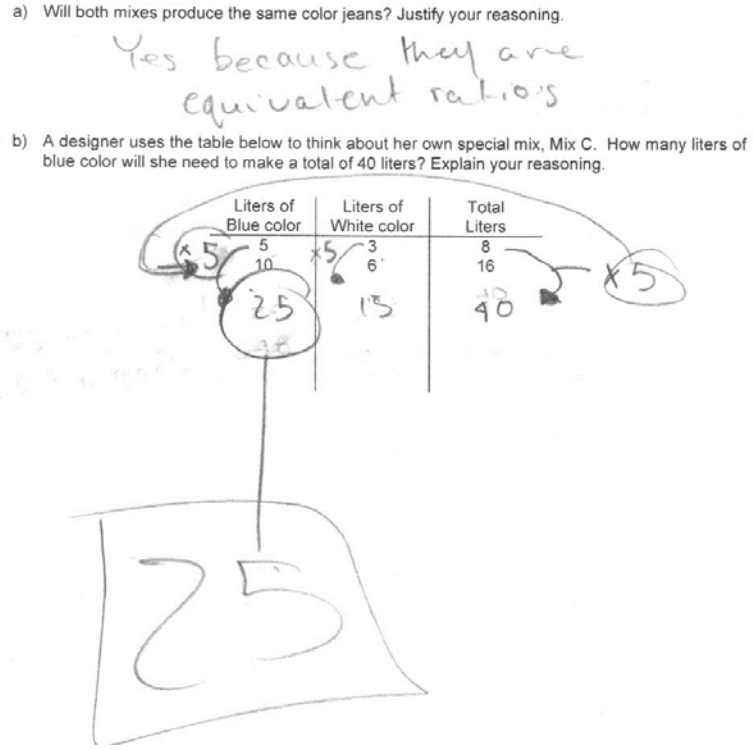
The response demonstrates adequate evidence of the learning and strategic tools necessary to complete the prompted purpose. It may contain overlooked issues, misleading assumptions, and/or errors in execution. Evidence in the response demonstrates that the student can revise the work to accomplish the task with the help of written feedback or dialogue.

Either verbally or symbolically, the strategy used to solve the problem is stated, as is the work used to find ratios, unit rates, scaling, and partial answers to problems. Reasoning may contain incomplete, ambiguous or misrepresentative ideas. Partial explanations are stated, based on work shown.

Accurate reasoning processes demonstrate the Mathematical Practice (1) Make sense of problems and persevere in solving them and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem with tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

Justification may include reasoning as follows:

a. Incorrectly forming <u>or</u> comparing unit rates or ratios for part a.
b. Building a table of values to a common number, but incorrectly interpreting results for part a.
c. Extending the table to 40 blue liters and/or a total of 40 liters, and possibly attempting to scale white liters to 40; therefore, incorrectly identifying the number of blue liters.
d. Using a proportion or proportional reasoning incorrectly (e.g., 40 blue liters is 5 times 8, so I can multiply 8 by 5 to find the total number of liters).


a) Will both mixes produce the same color jeans? Justify your reasoning.

Yes because they are equivalent ratios

b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
25	15	40

x5 x5 x5

25

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1 Point

The response demonstrates some evidence of mathematical knowledge that is appropriate to the intent of the prompted purpose. An effort was made to accomplish the task, but with little success. Evidence in the response demonstrates that, with instruction, the student can revise the work to accomplish the task.

Some evidence of reasoning is demonstrated either verbally or symbolically, is often based on misleading assumptions, and/or contains errors in execution. Some work is used to find ratios, unit rates, scaling or partial answers to portions of the task are evident. Explanations are incorrect, incomplete or not based on work shown. The student may make some attempt at using some mathematical practices. Accurate reasoning processes demonstrate the Mathematical Practice, (1) Make sense of problems and persevere in solving them, and (2) Reason abstractly and quantitatively (since students need to abstract information from the problem, create a mathematical representation of the problem, and correctly work with both part-part and part-whole situations). Evidence of the Mathematical Practice, (3) Construct viable arguments and critique the reasoning of others, is demonstrated by complete and accurate explanations. Evidence of the Mathematical Practice, (4) Model with mathematics, is demonstrated by representing the problem tables, ratios and/or proportions. Evidence of the Mathematical Practice, (6) Attend to precision, can include proper use of ratio and/or proportion notation, proper symbolism and proper labeling of quantities.

The reasoning used to solve the parts of the problem may include:

a. No attempt at mathematical reasoning to respond to part a.

b. Some attempt to scale in either or both parts, but failure to maintain the ratio, typically by reverting to addition.

c. Incorrectly forming <u>and</u> comparing unit rates or ratios for part a; also, scaling blue, white and total in part b.

I think NO because mix b is darker and lighter.

b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40
30	18	
35	21	
40	24	
	27	
	30	
	36	
	39	
	42	

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COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

GRADE 6 MATH: RATIOS AND PROPORTIONAL RELATIONSHIPS

NYC Department of Education
Dennis M. Walcott, Chancellor

Assessment 1: Question 1

1. Giovanni is visiting his grandmother who lives in an apartment building on the 24th floor. Giovanni enters the elevator in the lobby, which is the first floor in the building. The elevator stops on the 16th floor. What percentage of 24 floors does Giovanni have left to reach his grandmother's floor? Use pictures, tables or number sentences to solve the task. Explain your reasoning in words.

CCSS (Content) Addressed by this Task:

6.RP.1	Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3c	Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics (possible)
6. Attend to precision

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Student Work A

The percentage of floors, Giovanny have left is $33\frac{1}{3}$% because I simplified $\frac{16}{24}$ to $\frac{2}{3}$ $\frac{16}{24} = \frac{2}{3} = 66\frac{2}{3}$% which equals $66\frac{2}{3}$% and I subtracted 100 and $66\frac{2}{3}$ which gave me $33\frac{1}{3}$.

$$\begin{array}{r} 1\cancel{0}\cancel{0} \\ -66\frac{2}{3} \\ \hline 33\frac{1}{3} \end{array}$$

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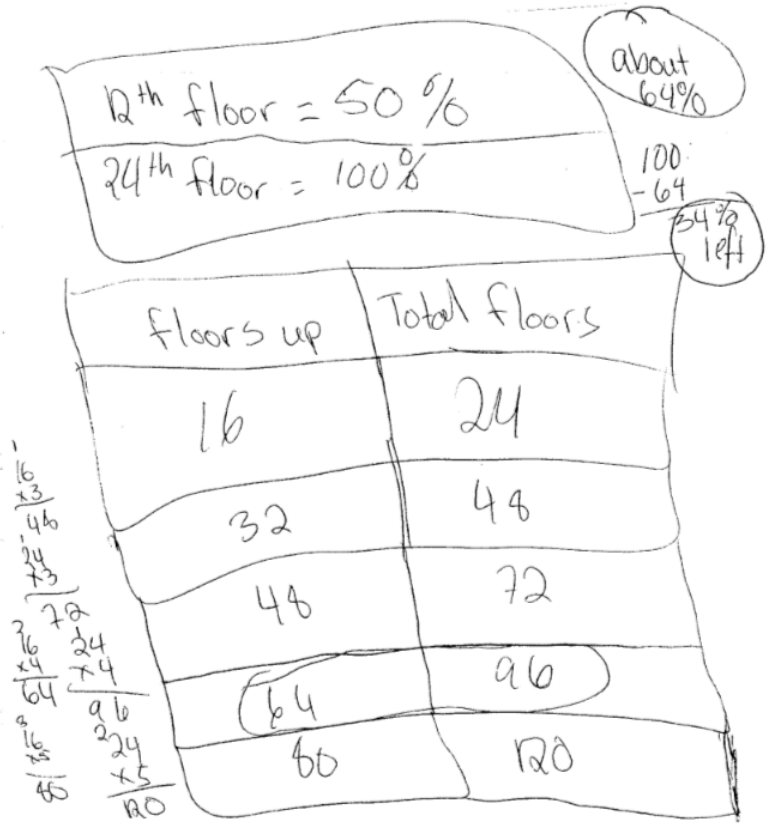
Student Work B



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Student Work C

24 = 100%

8%

12 = 50%

16 = 92%

I think it's 8% pacent because 24 would be the same 100%. Also from 16 to 24 is 8. So I think he has 8% to go.

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Assessment 1: Question 2

2. Pianos and pipe organs contain keyboards, a portion of which is shown below.

a) What is the ratio of black keys to white keys in the picture above?

b) If the pattern shown continues, how many black keys appear on a portable keyboard with 35 white keys?

c) If the pattern shown continues, how many black keys appear on a pipe organ with a total of 240 keys?

CCSS (Content) Addressed by this Task:

6.RP.1	Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2 (possible)	Understand the concept of a unit rate *a/b* associated with a ratio *a:b* with *b≠0*, and use rate language in the context of a ratio relationship.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a (potential)	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
4. Model with mathematics (possible)
6. Attend to precision
8. Look for and express regularity in repeated reasoning

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Student Work A

a)

W	B
5	7

5 : 7

b)

B	W
5	7
10	14
15	21
20	28
25	35

25 : 35

25 blank keys

c)

48 x 5 = 240

48 x 7 = 336

Black (10)	White (14)	Total
10	14	24
20	28	48
30	42	72
40	56	96
50	70	120
60	84	144
70	98	168
80	112	192
90	126	216
100	140	240

118
14
132
14
146

100 Black keys

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Student Work B

a)

The ratio of black keys to white keys is 5:7.

b)

There will be 25 black keys with 35 white keys.

$\frac{5}{7} = \frac{25}{35}$

c)

$6\frac{6}{7}$ black keys

$\frac{25}{35} = \frac{5}{7} = \frac{66}{240}$

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Student Work C

a) 5 black keys to 7 white keys

b) 33 black keys to 35 white keys

c) 238 black keys to 240 white keys

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Assessment 1: Question 3

3 a) Mr. Black's class has a female student-to-male student ratio of 3:2. If Mr. Black's class has 18 girls, how many boys does he have? Show or explain in writing how you determined your answer.

b) Ms. Green's class has the same number of students as Mr. Black's class. Her female-to-male ratio is 2:1. Which class has the greater number of females? How do you know?

CCSS (Content) Addressed by this Task:

6.RP.1	Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (possible)

CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
6. Attend to precision
7. Look for and express regularity in repeated reasoning

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Student Work A

3. a) 3:2 3 to 2 $\frac{3}{2}$

$\frac{3}{2} \times \frac{6}{6} = \frac{18}{12}$

12 boys I figured this by using an equivalent fractions since 3 times 6 is 18 which are the girls then 2 times 6 is 12 for boys

$\begin{array}{r} 12 \\ +\ 18 \\ \hline 30 \end{array}$

b) $\frac{2 \times 10}{1 \times 10} = \frac{20}{10}$ $\frac{2 \times 9}{1 \times 9} = \frac{18}{9}$

Ms Greens class has more females because since she has 30 students like Ms Black so Ms Black has $\frac{3}{2} = \frac{18}{12}$ girls to make 30 boys $\begin{array}{r} 18 \\ +\ 12 \\ \hline 30 \end{array}$

so then $\frac{2}{1} = \frac{20}{10}$ so Ms Green has more females

$\begin{array}{r} 20 \\ +\ 10 \\ \hline 30 \end{array}$

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Student Work B

3. a)

3	2
6	4
9	6
12	8
15	10
18	12

= Answer

18:12

$$\begin{array}{r} 18 \\ +12 \\ \hline 30 \end{array}$$

b)

2	1
4	2
6	3
8	4
10	5
12	6

12	6
14	7
16	8
18	9
20	10
22	11
24	12
26	13
28	14
30	15

30:15

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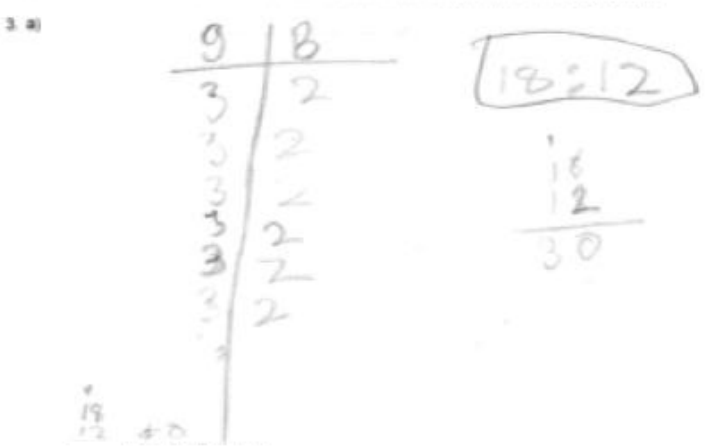
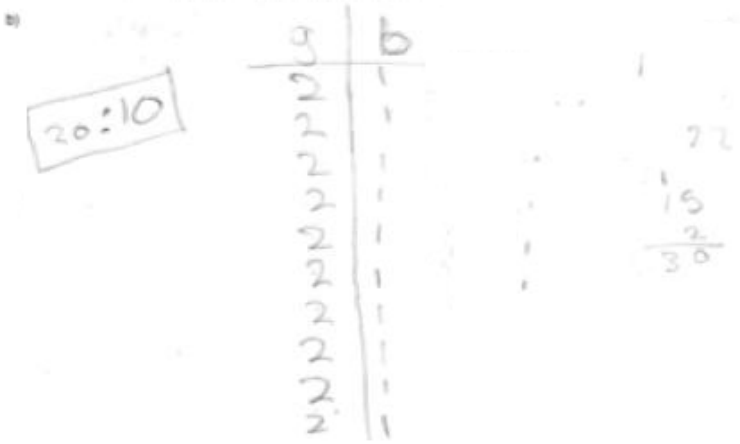
Student Work C





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Assessment 1: Question 4

4. Use the recipe shown in the table to answer the questions below. Use pictures, tables or number sentences to solve the task.

Grandma's Recipe for Sugar Cookies
1 ½ cups butter
2 cups sugar
4 eggs
¾ teaspoon baking powder
1 ¼ cups flour
¼ teaspoon salt

a) How many cups of sugar are needed for each egg? How do you know?

b) Your sister notices that she needs three times as much baking powder as salt in this recipe. What is the ratio of baking powder to salt? Explain how you know.

CCSS (Content) Addressed by this Task:

6.RP.1	Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3b	Solve unit rate problems including those involving unit pricing and constant speed.

CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics (possible)
6. Attend to precision
7. Look for and make use of structure.

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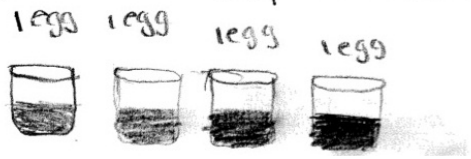
Student Work A

a) One half cup is needed for each egg.



b) there is a 3:1 ratio baking powder to salt. She said she needs three times as much
3/4 teaspoon baking powder is three times as much as 1/4 teaspoon salt.

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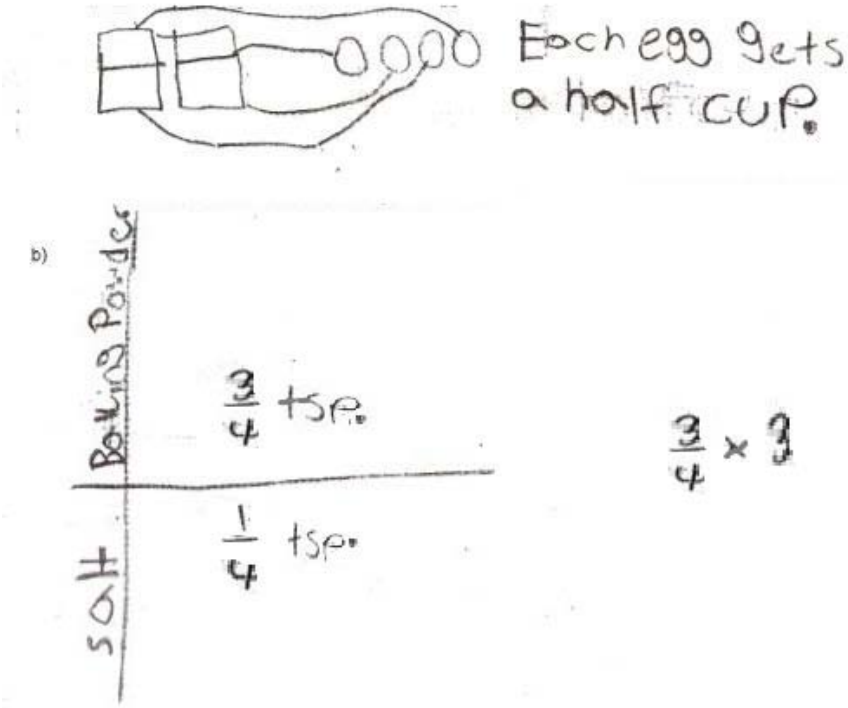
Student Work B



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Student Work C

a) 1 cup of shinger for each 2 eggs because 2÷2=1 4÷2=8

b) $1\frac{1}{2}+1\frac{1}{2}+1\frac{1}{2}=4\frac{1}{2}$

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$

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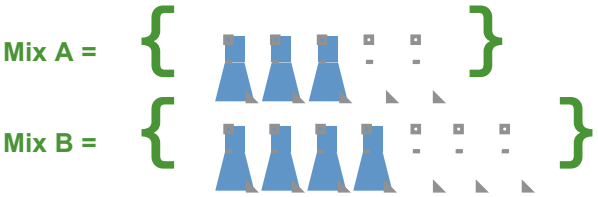
Assessment 1: Question 5

5. Fashion designers are trying to decide on just the right shade of blue for a new line of jeans. They have several bottles of fabric color, some with blue color and some with white color. They plan to mix these together to get the desired color.



a) Will both mixes produce the same color jeans? Justify your reasoning.

b) A designer uses the table below to think about her own special mix, Mix C. How many liters of blue color will she need to make a total of 40 liters? Explain your reasoning.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16

CCSS (Content) Addressed by this Task:

6.RP.1	Understand the concept of ratio and use ratio language to describe a ratio relationship between two quantities.
6.RP.2	Understand the concept of a unit rate a/b associated with a ratio $a:b$ with $b \neq 0$, and use rate language in the context of a ratio relationship.
6.RP.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g. by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
6.RP.3a	Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. (possible)
6.RP.3b	Solve unit rate problems including those involving unit pricing and constant speed.
6.RP.3c	Find a percent of a quantity as a rate per 100; solve problems involving finding the whole, given a part and the percent.

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CCSS for Mathematical Practice Addressed by this Task:

1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with Mathematics (possible)
6. Attend to precision
7. Look for and make use of structure

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Student Work A

5 a) 2:3x4=8:12

3:4x3=9:12

no mix B has more white then mix A

b)

Liters of Blue color	Liters of White color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40

When you use a graf if you contine the patern the mix will remane the same exsept the total will get biger

25

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Next Instructional Steps: Ask the student to explain what s/he did to scale up the ratios and how the factors were chosen. Press the student to represent what s/he actually did mathematically and to explain what happens when you multiply a ratio by 4/4 or 3/3. Ask the student to explain what s/he means by "Mix B has more white than Mix A". Play devil's advocate: "in the original picture, Mix B also has more white than Mix A. Why can't you just answer the question from what is shown there?"

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Student Work A

Mix A = { ▲▲▲□□ } $\frac{3}{2}$

Mix B = { ▲▲▲▲□□□ } $\frac{4}{3}$

a)

No, because Mix A = $\frac{3}{2}$ and Mix B = $\frac{4}{3}$.

b)

I divided 40 and 8 = 5 and 5X5=25, because 25.

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16
25	15	40

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Student Work B

a) yes. in both there is only one less in the whites to the blues

b)

Liters of Blue Color	Liters of White Color	Total Liters
5	3	8
10	6	16
15	9	24
20	12	32
25	15	40

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