COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

KINDERGARTEN MATH: BOOKS ON SHELVES PERFORMANCE TASK

NYC Department of Education
Dennis M. Walcott, Chancellor

Name_____

Books on Shelves

Miguel has two shelves. Miguel has six books. Miguel wants to put books on the two shelves. How many different ways can Miguel put books on the two shelves? Show and tell how you know.

KINDERGARTEN MATH:BOOKS ON SHELVES RUBRIC

The following section contains two rubrics that were used to score student work: a content rubric and a process rubric. The content rubric describes student performance according to the content standards in the CCLS. The process rubric describes student performance according to the National Council of Teachers of Mathematics (NCTM) process standards. Students' were given a score based on their achievement on the CCLS content rubric and the process rubric. Given that the process rubric is not in the language of the Common Core's Mathematical Practices, we have also included a document that NCTM has posted regarding the relationship between the NCTM process standards and the Standards for Mathematical Practice.

CCSS Mathematics Content Standards & Standards in Practice

Students apply mathematical reasoning, knowledge, and skills in problems-solving situations and support their solutions using mathematical language and appropriate representations (data).

Grades K-1 Progress-Monitoring Focus: Addition & subtraction; Comparing & ordering numbers

Gr. K-1 **Math CCSS Criteria/Clusters**	**Novice**	**Apprentice**	**Practitioner**	**Expert**
Counting & Cardinality (K only)	May recognize number symbols and names, but lacks counting sequence A numerical answer may be correct, but is not supported by student work (e.g., solves problem without applying properties of operations; just copies numbers) OR stated answer is incorrect or lacks relevance	Some parts of problem correct and those parts supported by student work Represents and solves simple addition and subtraction problems using counting, models, visuals, manipulatives, number lines, sounds, etc. May apply commutative property	***Represents number of objects for written numerals, 0-20*** **K.CC-3** ***Counts to find out how many; Determines greater than/less than of groups of objects (up to 20 if ordered; up to 10 if random)*** **K.CC- 4, 5** ***Compares numbers between 1 and 10 using written numerals*** **K.CC-6, 7**	Represents, compares, and solves problems using numbers greater than 20 Applies associative or commutative properties to solve problems

Operations & Algebraic Thinking **And** **Number & Operations in Base Ten**	Uses place value to show 10 or less	Represents and solves simple addition and subtraction problems using counting, models, visuals, manipulatives, number lines, sounds, etc. May apply commutative property	***Represents, compares, & solves addition & subtraction using strategies – counting, objects, drawings, etc. …*** ***-to 10*** **K.OA-1, 2** ***-or to 20; with multiple addends; using equal sign and equations*** **1. OA-1, 2, 6, 7, 8** ***Composes/ decomposes and compares numbers using tens and ones*** **K.OA-3; K.NBT-1** **1.NBT- 2, 3** ***Applies associative & commutative properties*** **1.OA-3** *Minor computation flaws do not affect outcome of a correct solution*	Represents, compares, and solves problems using numbers up to 100 (K) 120 (gr1) All parts of problem correct, precise, and supported by student work Applies associative or commutative properties to solve problems in more than one way Uses place value to expand numbers (gr 1)
Measurement & Data	Still demonstrates limited number sense (e.g., difficulty counting, estimating; representing quantities; recognizing measurement attributes,) Tells time to the hour using a digital clock	Recognizes and uses 1 measureable attribute to compare or classify Tells time to hour, & half hour (gr 1) using a digital clock	***Describes measurable attributes; compares measures of objects*** **K.MD-1, 2** ***Measures to whole unit; compares 2 or 3 objects; classifies objects using more than 1 attribute*** **1.MD-1, 2** ***Measures & compares lengths indirectly*** **1.MD-1** ***Tells time to hour & half hour using digital and analog clocks*** **1.MD-3**	Uses a variety of strategies to estimate, measure, and compare Measures and compares lengths of more than 2 objects and determines the difference in lengths

Exemplars® Standards-Based Math Rubric*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Novice	No strategy is chosen, or a strategy is chosen that will not lead to a solution. Little or no evidence of engagement in the task present.	Arguments are made with no mathematical basis. No correct reasoning nor justification for reasoning is present.	No awareness of audience or purpose is communicated. or Little or no communication of an approach is evident or Everyday, familiar language is used to communicate ideas.	No connections are made.	No attempt is made to construct mathematical representations.
Apprentice	A partially correct strategy is chosen, or a correct strategy for only solving part of the task is chosen. Evidence of drawing on some previous knowledge is present, showing some relevant engagement in the task.	Arguments are made with some mathematical basis. Some correct reasoning or justification for reasoning is present with trial and error, or unsystematic trying of several cases.	Some awareness of audience or purpose is communicated, and may take place in the form of paraphrasing of the task. or Some communication of an approach is evident through verbal/written accounts and explanations, use of diagrams or objects, writing, and using mathematical symbols. or Some formal math language is used, and examples are provided to communicate ideas.	Some attempt to relate the task to other subjects or to own interests and experiences is made.	An attempt is made to construct mathematical representations to record and communicate problem solving.

*Based on revised NCTM standards.

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Exemplars® Standards-Based Math Rubric (cont.)*

	Problem Solving	Reasoning and Proof	Communication	Connections	Representation
Practitioner	A correct strategy is chosen based on mathematical situation in the task. Planning or monitoring of strategy is evident. Evidence of solidifying prior knowledge and applying it to the problem solving situation is present. Note: The practitioner must achieve a correct answer.	Arguments are constructed with adequate mathematical basis. A systematic approach and/or justification of correct reasoning is present. This may lead to... • clarification of the task. • exploration of mathematical phenomenon. • noting patterns, structures and regularities.	A sense of audience or purpose is communicated. and/or Communication of an approach is evident through a methodical, organized, coherent sequenced and labeled response. Formal math language is used throughout the solution to share and clarify ideas.	Mathematical connections or observations are recognized.	Appropriate and accurate mathematical representations are constructed and refined to solve problems or portray solutions.
Expert Work at this level is exceeding grade-level expectations	An efficient strategy is chosen and progress towards a solution is evaluated. Adjustments in strategy, if necessary, are made along the way, and / or alternative strategies are considered. Evidence of analyzing the situation in mathematical terms, and extending prior knowledge is present. Note: The expert must achieve a correct answer.	Deductive arguments are used to justify decisions and may result in formal proofs. Evidence is used to justify and support decisions made and conclusions reached. This may lead to... • testing and accepting or rejecting of a hypothesis or conjecture. • explanation of phenomenon. • generalizing and extending the solution to other cases.	A sense of audience and purpose is communicated. and/or Communication at the Practitioner level is achieved, and communication of argument is supported by mathematical properties. Precise math language and symbolic notation are used to consolidate math thinking and to communicate ideas.	Mathematical connections or observations are used to extend the solution.	Abstract or symbolic mathematical representations are constructed to analyze relationships, extend thinking, and clarify or interpret phenomenon.

*Based on revised NCTM standards.

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National Council of Teachers of Mathematics (NCTM) Process Standards and the Common Core State Standards for Mathematics

From ***NCTM Action on the Common Core State Standards for Mathematics***

by NCTM President J. Michael Shaughnessy

"The preeminent message in both the NCTM *Principles and Standards for School Mathematics* (2000) and CCSSM is the importance of nurturing mathematical thinking and reasoning processes in our students. No bulleted list of specific content standards will hold together as a coherent, meaningful whole, or make any significant contribution to our students' growth in mathematics, without interweaving mathematical "practices." Mathematics curricula must show students the power of reasoning and sense making as they explore mathematical structures, of communication as they construct viable arguments, and of multiple representations as they engage in mathematical modeling. The close connections between the NCTM Process Standards and the CCSSM Standards for Mathematical Practice are represented in the chart below.

The upcoming NCTM publication, *Making it Happen*, will provide a deeper analysis of the connections between the NCTM Process Standards and detail the potential of the existing NCTM resources to interpret and implement CCSSM."

NCTM Process Standards and the CCSS Mathematical Practices

NCTM Process Standards	CCSS Standards for Mathematical Practice
Problem Solving	1. Make sense of problems and persevere in solving them. 5. Use appropriate tools strategically.
Reasoning and Proof	2. Reason abstractly and quantitatively. 3. Critique the reasoning of others. 8. Look for and express regularity in repeated reasoning
Communication	3. Construct viable arguments
Connections	6. Attend to precision. 7. Look for and make use of structure
Representations	4. Model with mathematics.

COMMON CORE-ALIGNED TASK
WITH INSTRUCTIONAL SUPPORTS

Mathematics

KINDERGARTEN MATH: BOOKS ON SHELVES ANNOTATED STUDENT WORK

This section contains annotated student work at a range of score points, student summaries, and implications for instruction for each performance level. The annotated student work and student summaries demonstrate performance at different levels and show examples of student understandings and misunderstandings of the task that can be used with the implications for instruction to understand how to move students to the next performance level.

NYC Department of Education
Dennis M. Walcott, Chancellor

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Expert according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Expert

Name Student 1

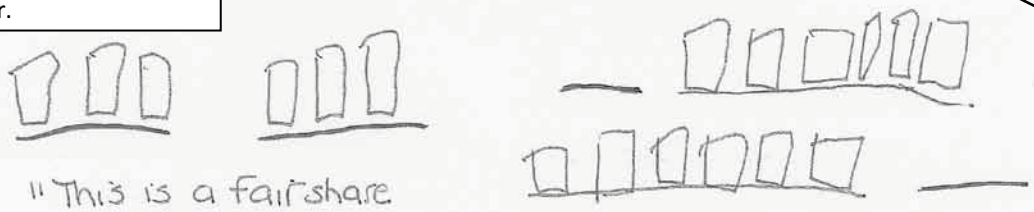
Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student uses a diagram to show the five possible combinations for six books on two shelves. The student states a correct answer, "5 ways."

The student models with mathematics by creating a diagram and a key, and a table with correct labels and data. The student uses both representations to clarify and support the answer.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).



"This is a fair share of books for the two shelves. That means equal. 6 books means even number."

combinations

"This word is combinations. Miguel has 5 of 'em."

"If the books can go on only one shelf you can have two more ways - zero and six six and zero

"I think I can do a table."

page 1 of 2

The student applies the precise mathematical language- diagram, key, fair share, equal, even number, combinations, table, pattern, to help construct viable arguments.

The student constructs a new argument, "If the books can go on only one shelf you can have two more combinations- zero and six, six and zero."

Kindergarten Math: Books on Shelves
Annotated Student Work

The student justifies the reasonableness of her/his conclusions by verifying the answer using a new strategy of a table and states, "Tell Miguel it is five ways cuz I know it is." The student searches for regularity and discerns a pattern, "Look the books go up by 1 here and books go down by one here. That is a pattern."

books shelf 1	books shelf 2
1	5
2	4
3	3
4	2
5	1

1+5=6
2+4=6
3+3=6
4+2=6
5+1=6
5

"Look, the books go up by 1 here (pointed to shelf 1) and books go down by 1 here (pointed to shelf 2) that is a pattern.
I did 5 ways here and here (pointed to diagram and table). Tell Miguel it is 5 ways cuz I know it is." A2

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Kindergarten Math: Books on Shelves
Annotated Student Work

Expert – Student 1 Summary

Achievement Level: Student 1 is an Expert according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Expert	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5 ways." The student uses an alternative strategy of a table to support her/his answer.
Reasoning and Proof Expert	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem. The student verifies her/his solution by solving the problem a second way.
Communication Expert	The student correctly uses the mathematical terms-diagram, key, fair share, equal, even number, combinations, table, pattern.
Connections Expert	The student makes the Practitioner mathematically relevant connections, "This is a fair share of books for the two shelves," "That means equal," "Six books means even number," "If the books can go on only one shelf you can have two more ways-zero and six, six and zero," and, "Look the books go up by one here and books go down by one here, that is a pattern." The student makes the Expert connection by using a table to solve the problem a second way and linking the two strategies together to verify that her/his solution is correct. The student states, I did five ways here and here. Tell Miguel it is five ways cuz I know it is."
Representation Expert	The student's diagrams are appropriate to the problem and accurate. The student's key and the scribing correctly define the books and shelves. The student's table is appropriate to the problem and accurate. Each column is correctly labeled and all entered data is correct. The student uses her/his table to clarify and support the answer.

Instructional Implications: Books on Shelves K

Achievement Level: Expert

Note: Student work identified at this level is exceeding grade-level expectations

The following is a list of instructional implications that you may want to consider for students performing at the Expert level. In addition, you may want to consult the suggestions for the Practitioner level:

- Find combinations for numbers greater than ten
- Investigate and prove generalization-total combinations is one less than sum being found
- Investigate and prove generalization-if zero can be considered the total combinations is one more than sum
- Solve problem more than one way to verify that answer is correct
- Relate problem to a similar one completed and discuss how they are mathematically similar
- Discover and discuss why an even number has a fair share/equal combination but an odd number does not

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

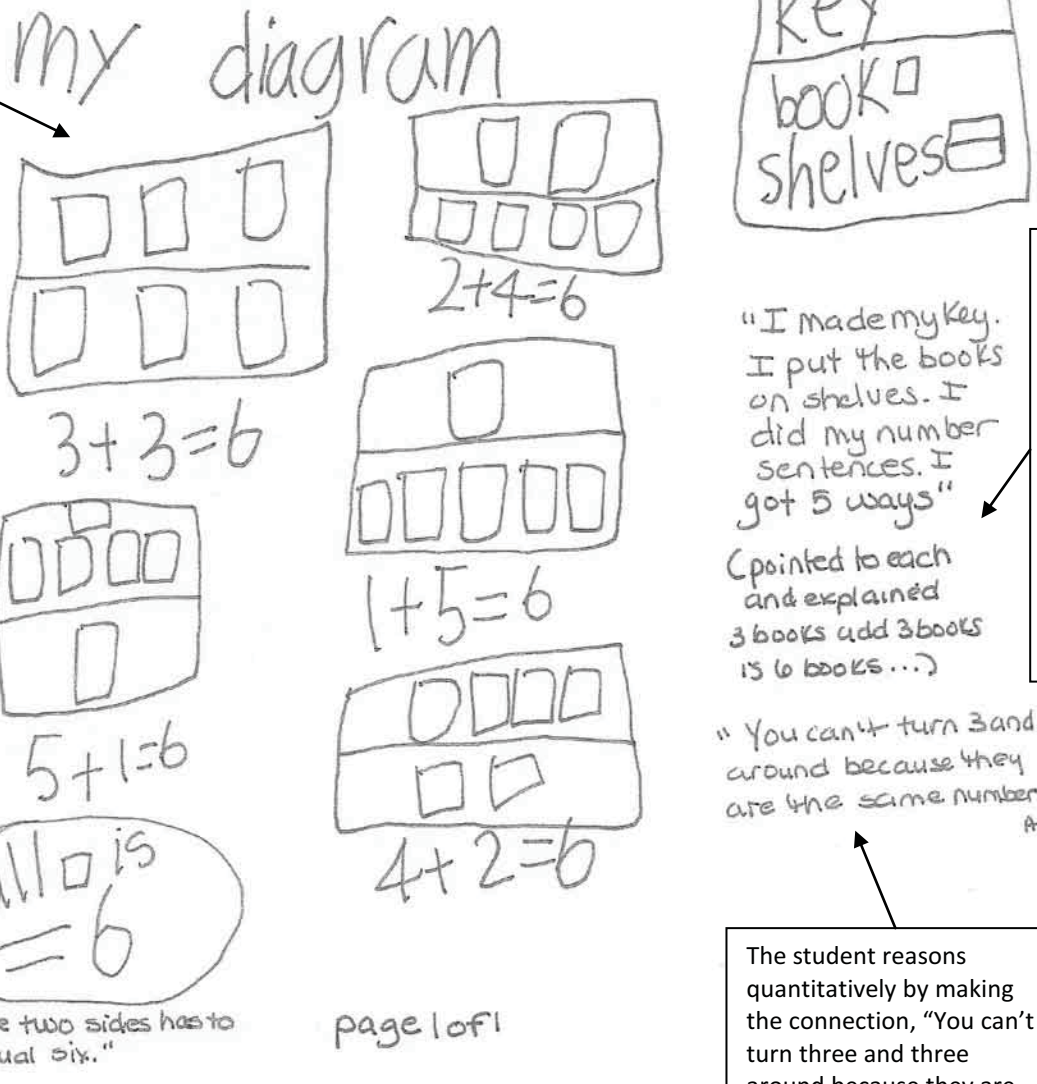
Name Practitioner Student 1 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student applies the precise mathematical language-diagram, key, number sentences, equal, to help construct her/his viable argument.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).



The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student shows the five possible combinations for six books on two shelves and correctly states, "I got five ways."

The student models mathematics by creating a diagram that is appropriate and accurate. A key and scribing correctly defines the books and shelves.

The student reasons quantitatively by making the connection, "You can't turn three and three around because they are the same number."

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner – Student 1 Summary

Achievement Level: Student 1 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram and number sentences to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "I got five ways."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key, number sentences, equal.
Connections Practitioner	The student makes the mathematically relevant connection, "You can't turn three and three around because they are the same number." The student's statements, "all □ is + 6," and, "The two sides has to equal six," are not considered connections as they represent the reasoning a student must understand to solve the problem.
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key and the scribing correctly defines the books and shelves.

Kindergarten Math: Books on Shelves
Annotated Student Work

Exemplars Rubric: Practitioner
CCSS Content Rubric: Expert

This student is a Practitioner according to the Exemplars Rubric and an Expert according to the CCSS Content Standards Rubric, (both included in the supporting materials).

The student is able to make sense of the problem and persevere in solving it. The student shows the five possible combinations for six books on two shelves, and correctly states, "Miguel has five ways to do the books."

Name Student 2 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student applies the precise mathematical language-table, key, to help construct her/his viable argument.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

"I did a table
I made a key
for shelf 1 and 2
and 6 books
I went 1 book, 2 book
3 book, 4 book, 5 book
so I don't miss any
ways. We learned that
plan. Miguel has 5
ways to do the books.
1 plus 5, 2 plus 4, 3 plus 3
4 plus 2, 5 plus 1 (pointed to table)
They all end at 6 books."

Table 2

1	2
1	5
2	4
3	3
4	2
5	1

"I did it with
the numbers
It is faster
this way."

"3 books and 3 books
is the same on
shelf 1 and shelf 2"

AZ

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The student reasons abstractly and quantitatively by making the connection, "3 books and 3 books is the same on shelf one and shelf two and searches for regularity by stating, "I went 1 book, 2 book, 3 book, 4 book, 5 book, so I don't miss any ways.

The student models with mathematics by using a table with a key to define the shelves and books. The student reflects and improves the model/table by keeping the key for shelves but uses numbers in place of the books and states, "I did it with numbers. It is faster this way."

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner/Expert – Student 2 Summary

Achievement Level: Student 2 is a Practitioner according to the Exemplars Process Rubric and an Expert according to the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a table to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and states, "5," which is also correct. The scribing also correctly states, "Miguel has five ways to do the books. One plus five, two plus four, three plus three, four plus two, five plus one."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-table, key.
Connections Practitioner	The student makes the mathematically relevant connection of using a different form of a table to show the five correct combinations. The student states, "I did it with numbers. It is faster this way." The student makes the mathematically relevant observation, "Three books and three books is the same on shelf one and shelf 2."
Representation Practitioner	The student's first table is appropriate to the problem and accurate. The key, column labels, and scribing correctly identify the books and shelves.

Kindergarten Math: Books on Shelves
Annotated Student Work

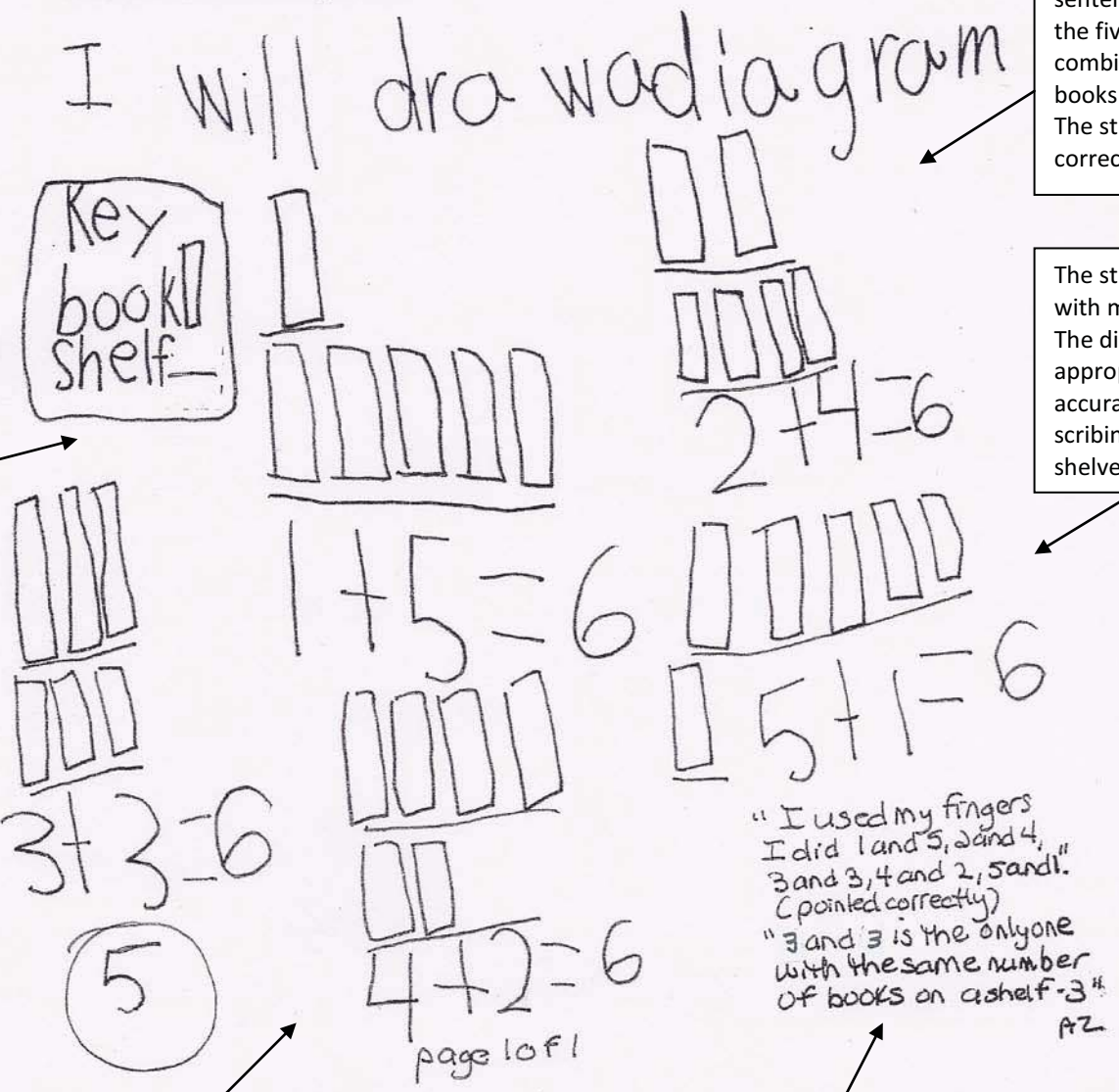
This student is a Practitioner according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Name Practitioner Student 3C

K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.



The student is able to make sense of the problem and persevere in finding a pathway to solving it. The student uses a diagram and number sentences to show the five possible combinations for six books on two shelves. The student states a correct answer, "5."

The student applies the precise mathematical language-diagram, key, to help construct her/his viable argument.

The student models with mathematics. The diagram is appropriate and accurate. A key and scribing define the shelves and books.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

The student reasons abstractly and quantitatively by making the connection, "three and three is the only one with the same number books on a shelf-three."

Kindergarten Math: Books on Shelves
Annotated Student Work

Practitioner – Student 3 Summary

Achievement Level: Student 3 is a Practitioner according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram and number sentences to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5." The student also correctly states, "I used my fingers. I did one and five, two and four, three and three, four and two, five and one."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 can not be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key
Connections Practitioner	The student makes the mathematically relevant connection, "six and six is the only one with the same number of books on a shelf-three."
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key and the scribing correctly label the books and shelves.

Instructional Implications: Books on Shelves K

Student Achievement Level: Practitioner

The following is a list of instructional implications that you may want to consider for students performing at the Practitioner level. In addition, you may want to consult the suggestions for the Novice and Apprentice levels:

- Include more writing of number sentences to support the possible combinations for any number up to ten in a student's diagram, table, etc.
- Discuss and solve problems where zero would be used in a combination and when it would not be used
- Introduce the language term-commutative property
- Encourage student to make more than one mathematically relevant connection about her/his work
- Investigate where a combination is a fair share
- Introduce another strategy to solve the same problem-model, diagram/key, organized list, table

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Apprentice

Name Student 1

K

Books on Shelves

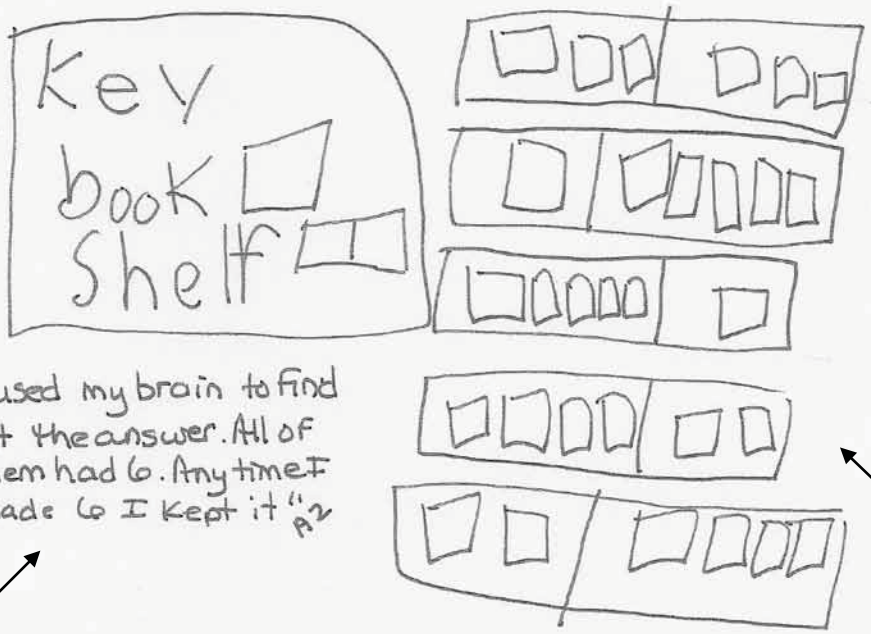
Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student is able to make sense of the problem and persevere in finding a solution pathway. The student's answer, "5," is correct.

make a diagram.

5

The student applies the precise mathematical language- diagram, key, to help construct her/his viable argument.



The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

"I used my brain to find out the answer. All of them had 6. Any time I made 6 I kept it" A2

The student models with mathematics. The diagram is appropriate and accurate. A key defines the books and shelves.

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

there is 6 book

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Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice - Student 1 Summary

Achievement Level: Student 1 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Practitioner	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves works to solve the problem. The student shows five correct combinations and correctly states, "5."
Reasoning and Proof Practitioner	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, the commutative property, and understanding that 0 + 6 = 6 and 6 + 0 = 6 cannot be considered as possible combinations for the problem.
Communication Practitioner	The student correctly uses the mathematical terms-diagram, key.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Practitioner	The student's diagram is appropriate to the problem and accurate. The student's key correctly defines the books and shelves.

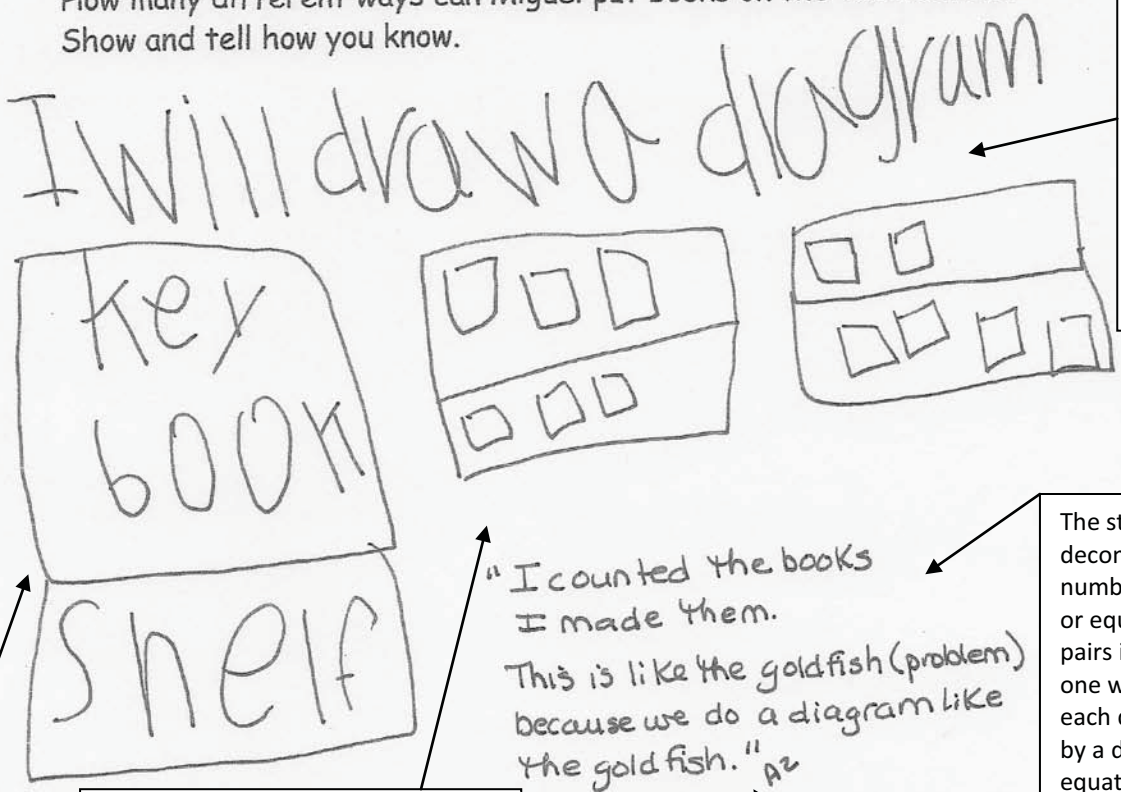
Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Name Apprentice Student 2 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.



"I counted the books I made them. This is like the goldfish (problem) because we do a diagram like the gold fish." A2

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The student is able to make sense of the problem but does not persevere in solving it. The student only finds two possible combinations.

The student applies the precise mathematical language- diagram, to help construct her/his viable argument. The term, key, is not credited because the key is not accurate.

The student models with mathematics making a diagram that is appropriate but not accurate. The key is not complete and the student's scribing only defines the books.

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3), but only does so for two possible combinations for six.

Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice – Student 2 Summary

Achievement Level: Student 2 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student's strategy of using a diagram to show the total combinations for placing six books on two shelves would work to solve the problem. The student shows two correct combinations which is a partially correct answer.
Reasoning and Proof Apprentice	The student shows correct reasoning of the underlying mathematical concept of combinations to six but only completes two combinations. The student is missing 1 + 5 = 6, 4 + 2 = 6, and, 5 + 1 = 6.
Communication Apprentice	The student correctly uses the mathematical term-diagram. The student does not earn credit for the term-key because the student does not complete her/his key to demonstrate understanding of the term.
Connections Apprentice	The student attempts to make a mathematically relevant connection about her/his solution. The student starts to compare the Books on Shelves problem to the Goldfish problem but does not make a mathematical link about the common underlying mathematics of the two problems. The student needs to explain how the diagrams are similar and why.
Representation Apprentice	The student's diagram is appropriate to the problem but is not accurate. The student does not define the □ 's and does not include all five possible combinations.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is an Apprentice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Apprentice

Name Student 3 K

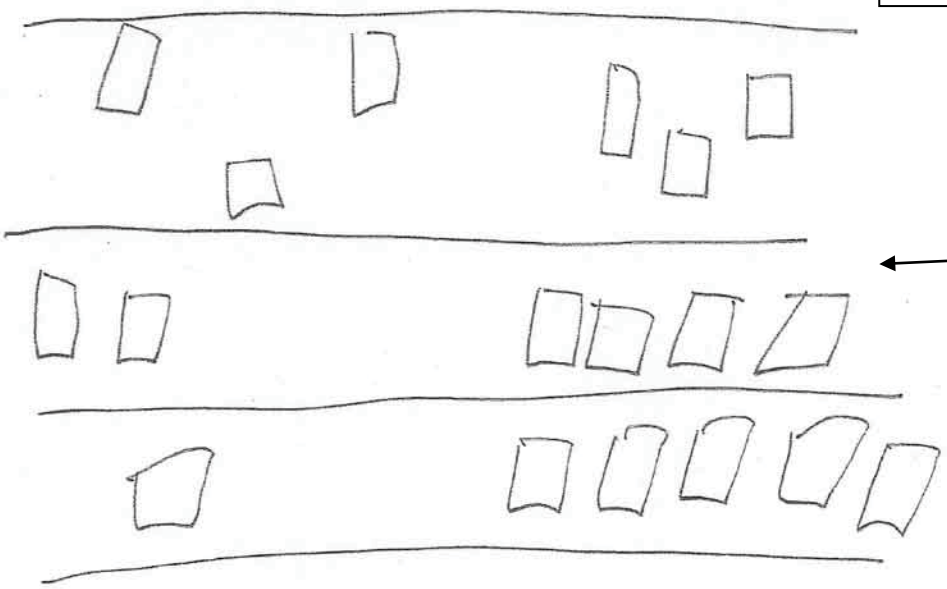
Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

The student applies the precise mathematical language- diagram to help construct her/his viable argument.

The student is able to make sense of the problem but does not persevere in solving it. The student only finds three possible combinations for six books on two shelves which leads to a partial answer.

My diagram



The student's diagram is appropriate but not accurate. Two combinations are not indicated and a key or scribing is not included to define the shelves and books.

3 ways

The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends.

The student can decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3), but only shows three possible combinations.

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Kindergarten Math: Books on Shelves
Annotated Student Work

Apprentice – Student 3 Summary

Achievement Level: Student 3 is an Apprentice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Apprentice	The student's strategy of using a diagram to show combinations for placing six books on two shelves would work to solve the problem. The student only shows three correct combinations. The student's answer, "3 ways," is not correct.
Reasoning and Proof Apprentice	The student shows correct reasoning of the underlying mathematical concepts of combinations to six, and the understanding that 0 + 6 = 6 and 6 + 0 = 6 cannot be considered as possible combinations for the problem. The student does not apply the commutative property in finding the combinations for 4 + 2 = 6 and 5 + 1 = 6
Communication Apprentice	The student correctly uses the mathematical term-diagram.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student's diagram is appropriate to the problem but is not accurate. The student does not define the books and shelves in a key or in any scribing. The diagram is missing the combinations for 4 and 2 and 5 and 1.

Instructional Implications: Books on Shelves K

Student Achievement Levels: Novice and Apprentice

The following is a list of instructional implications that you may want to consider for students performing at the Novice and Apprentice levels:

- continue to subitize with student to "see" sixness, or any number up to ten using hands and dot cards
- use manipulatives to investigate "fact families"/combinations of six or other numbers up to ten to discover how applying the commutative property in an organized manner can support a correct answer.
- use a ten frame to find combinations of six or other numbers up to ten
- use games such as cup and counters to find combinations of six or other numbers to up ten
- use graph paper and two color crayons to show combinations to six or other numbers up to ten, cut apart to show how the two "staircases" match for commutative property
- use number sentences to represent combinations/commutative property
- review mathematical language-model, number sentence, diagram, key, per, total, equal, add, fair share, combination, more than, less than
- review how to make a model with manipulatives or a diagram with a key
- have centers available for investigation and practice
- Provide leading questions to begin reflection on the solution in order to see regularities, structures, patterns, trends, etc.
- Solve similar problems using four, five, seven, eight, nine, or ten

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Novice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

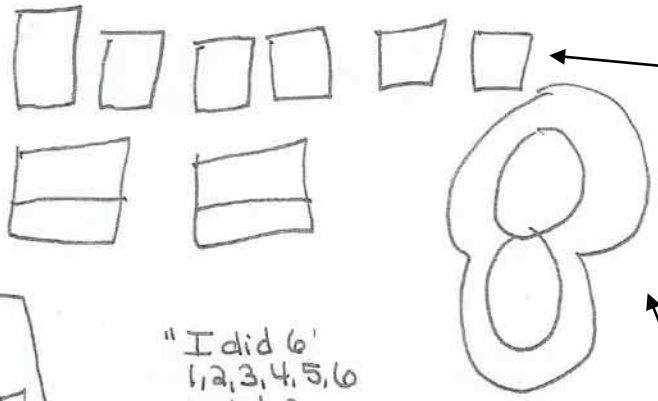
Name Novice Student 1 K

Books on Shelves

Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

I will draw a diagram
Miguel has two shelves

The student does not discern that the problem requires decomposing six books to find five possible combinations. It appears that the student adds the two shelves and six books for a total of "8" which is an incorrect answer.



The student does not decompose numbers less than or equal to 10 into pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).

Key
books □

"I did 6'
1,2,3,4,5,6
I did 2
1,2
I got 8
1,2,3,4,5,6,7,8"
A-2

The student applies the precise mathematical language-diagram and key, to help construct her/his argument.

The student attempts to model with mathematics but the diagram is not accurate.

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Kindergarten Math: Books on Shelves
Annotated Student Work

Novice - Student 1 Summary

Achievement Level: Student 1 is a Novice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student's strategy of using a diagram to show six books and what appears to be two shelves, adding the two numbers for a total of eight would not work to solve the problem. The student's answer, "8," is not correct.
Reasoning and Proof Novice	The student shows no correct reasoning of the underlying mathematical concepts of combinations, applying zero to combinations, and the commutative property. It seems that the student is simply adding the two numbers stated in the problem for a sum of eight.
Communication Practitioner	The student correctly uses the mathematical terms-diagram and key to support her/his thinking.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student attempts a diagram of books and shelves but the diagram is not accurate. The student is diagramming the six books and two shelves stated in the problem and adding the two numbers together.

Kindergarten Math: Books on Shelves
Annotated Student Work

This student is a Novice according to both the Exemplars Rubric and the CCSS Content Standards Rubric, (both included in the supporting materials).

Novice

Name Student 2

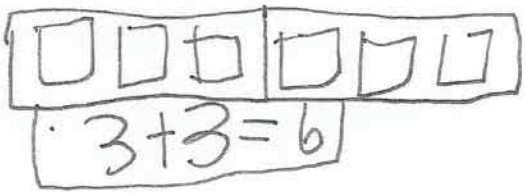
K

Books on Shelves

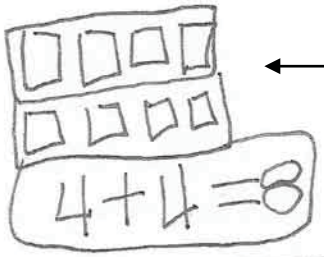
Miguel has two shelves.
Miguel has six books.
Miguel wants to put books on the two shelves.
How many different ways can Miguel put books on the two shelves?
Show and tell how you know.

I will draw a diagram

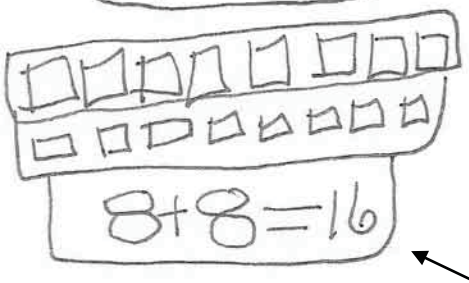
The student does not discern that the problem requires decomposing six books to find five possible combinations. The student states a correct combination for six but does not apply the concept of combinations of six to her/his solution.



The student does not decompose numbers less than or equal to 10 in pairs in more than one way, and record each decomposition by a drawing or equation (K.OA 3).



The student applies the precise mathematical language-diagram, key, to help construct her/his argument.



The student does not reflect on her/his solution to make connections to regularities, patterns, structures, or trends in the problem.

The student attempts to model with mathematics but the diagram is not accurate. The student is representing books and shelves to find the double of three, four, and eight.

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Kindergarten Math: Books on Shelves
Annotated Student Work

Novice - Student 2 Summary

Achievement Level: Student 2 is a Novice according to both the Exemplars Process Rubric and the CCSS Content Standards Rubric, (both of which are included in the supporting materials). The table below provides a rationale for the student's performance level in each of the criteria identified in the Exemplars Process Rubric.

Criteria and Performance Level	Rationale
Problem Solving Novice	The student's strategy of using a diagram to show three books and three books equaling six books, four books and four books equalling eight books, and eight books and eight books equalling sixteen books would not work to solve the problem. Although the student states a correct combination for six, the student is not applying the concept of combinations to her/his solution.
Reasoning and Proof Novice	The student shows no correct reasoning of the underlying mathematical concept of combinations to six. It appears that the student is finding "doubles" using three, four, and eight.
Communication Practitioner	The student correctly uses the mathematical terms-diagram and key to support her/his thinking.
Connections Novice	The student does not make a mathematically relevant connection about her/his solution.
Representation Apprentice	The student attempts a diagram of books and shelves but the diagram is not accurate. The student is using books and shelves to find the double of three, four, and eight instead of diagramming possible combinations of six.